3

STRUCTURAL ANALYSIS

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This chapter presents two examples that focus on the dynamic analysis of steel frame structures:

1. A 12-story steel frame building in Stockton, California – The highly irregular structure is analyzed using three techniques: equivalent lateral force (ELF) analysis, modal-response-spectrum analysis, and modal time-history analysis. In each case, the structure is modeled in three dimensions, and only linear elastic response is considered. The results from each of the analyses are compared, and the accuracy and relative merits of the different analytical approaches are discussed.

2. A six-story steel frame building in Seattle, Washington. This regular structure is analyzed using both linear and nonlinear techniques. Due to limitations of available software, the analyses are performed for only two dimensions. For the nonlinear analysis, two approaches are used: static pushover analysis in association with the capacity-demand spectrum method and direct time-history analysis. In the nonlinear analysis, special attention is paid to the modeling of the beam-column joint regions of the structure. The relative merits of pushover analysis versus time-history analysis are discussed.

Although the Seattle building, as originally designed, responds reasonably well under the design ground motions, a second set of time-history analyses is presented for the structure augmented with added viscous fluid damping devices. As shown, the devices have the desired effect of reducing the deformation demands in the critical regions of the structure.

Although this volume of design examples is based on the 2000 *Provisions*, it has been annotated to reflect changes made to the 2003 *Provisions*. Annotations within brackets, [ ], indicate both organizational changes (as a result of a reformat of all of the chapters of the 2003 *Provisions*) and substantive technical changes to the 2003 *Provisions* and its primary reference documents. While the general concepts of the changes are described, the design examples and calculations have not been revised to reflect the changes to the 2003 *Provisions*.

A number of noteworthy changes were made to the analysis requirements of the 2003 *Provisions*. These include elimination of the minimum base shear equation in areas without near-source effects, a change in the treatment of P-delta effects, revision of the redundancy factor, and refinement of the pushover analysis procedure. In addition to changes in analysis requirements, the basic earthquake hazard maps were updated and an approach to defining long-period ordinates for the design response spectrum was developed. Where they affect the design examples in this chapter, significant changes to the 2003 *Provisions* and primary reference documents are noted. However, some minor changes to the 2003 *Provisions* and the reference documents may not be noted.

In addition to the 2000 *NEHRP Recommended Provisions* (herein, the *Provisions*), the following documents are referenced:

AISC Seismic	American Institute of Steel Construction. 1997 [2002]. *Seismic Provisions for Structural Steel Buildings*.
ATC-40	Applied Technology Council. 1996. *Seismic Evaluation and Retrofit of Concrete Buildings*.
Bertero	Bertero, R. D., and V.V. Bertero. 2002. "Performance Based Seismic Engineering: The Need for a Reliable Comprehensive Approach," *Earthquake Engineering and Structural Dynamics* 31, 3 (March).
Chopra 1999	Chopra, A. K., and R. K. Goel. 1999. *Capacity-Demand-Diagram Methods for Estimating Seismic Deformation of Inelastic Structures: SDF Systems*. PEER-1999/02. Berkeley, California: Pacific Engineering Research Center, College on Engineering, University of California, Berkeley.
Chopra 2001	Chopra, A. K., and R. K. Goel. 2001. *A Modal Pushover Procedure to Estimate Seismic Demands for Buildings: Theory and Preliminary Evaluation*, PEER-2001/03. Berkeley, California: Pacific Engineering Research Center, College on Engineering, University of California, Berkeley.
FEMA 356	American Society of Civil Engineers. 2000. *Prestandard and Commentary for the Seismic Rehabilitation of Buildings*.
Krawinkler	Krawinkler, Helmut. 1978. "Shear in Beam-Column Joints in Seismic Design of Frames," *Engineering Journal*, Third Quarter.

3.1 IRREGULAR 12-STORY STEEL FRAME BUILDING, STOCKTON, CALIFORNIA

3.1.1 Introduction

This example presents the analysis of a 12-story steel frame building under seismic effects acting alone. Gravity forces due to live and dead load are not computed. For this reason, member stress checks, member design, and detailing are not discussed. For detailed examples of the seismic-resistant design of structural steel buildings, see Chapter 5 of this volume of design examples.

The analysis of the structure, shown in Figures 3.1-1 through 3.1-3, is performed using three methods:

1. Equivalent lateral force (ELF) procedure based on the requirements of *Provisions* Chapter 5,

2. Three-dimensional, modal-response-spectrum analysis based on the requirements of *Provisions* Chapter 5, and

3. Three-dimensional, modal time-history analysis using a suite of three different recorded ground motions based on the requirements of *Provisions* Chapter 5.

In each case, special attention is given to applying the *Provisions* rules for orthogonal loading and accidental torsion. All analyses were performed using the finite element analysis program SAP2000 (developed by Computers and Structures, Inc., Berkeley, California).

3.1.2 Description of Structure

The structure is a 12-story special moment frame of structural steel. The building is laid out on a rectangular grid with a maximum of seven 30-ft-wide bays in the X direction, and seven 25-ft bays in the Y direction. Both the plan and elevation of the structure are irregular with setbacks occurring at Levels 5 and 9. All stories have a height of 12.5 ft except for the first story which is 18 ft high. The structure has a full one-story basement that extends 18.0 ft below grade. Reinforced 1-ft-thick concrete walls form the perimeter of the basement. The total height of the building above grade is 155.5 ft.

Gravity loads are resisted by composite beams and girders that support a normal weight concrete slab on metal deck. The slab has an average thickness of 4.0 in. at all levels except Levels G, 5, and 9. The slabs on Levels 5 and 9 have an average thickness of 6.0 in. for more effective shear transfer through the diaphragm. The slab at Level G is 6.0 in. thick to minimize pedestrian-induced vibrations, and to support heavy floor loads. The low roofs at Levels 5 and 9 are used as outdoor patios, and support heavier live loads than do the upper roofs or typical floors.

At the perimeter of the base of the building, the columns are embedded into pilasters cast into the basement walls, with the walls supported on reinforced concrete tie beams over piles. Interior columns are supported by concrete caps over piles. All tie beams and pile caps are connected by a grid of reinforced concrete grade beams.

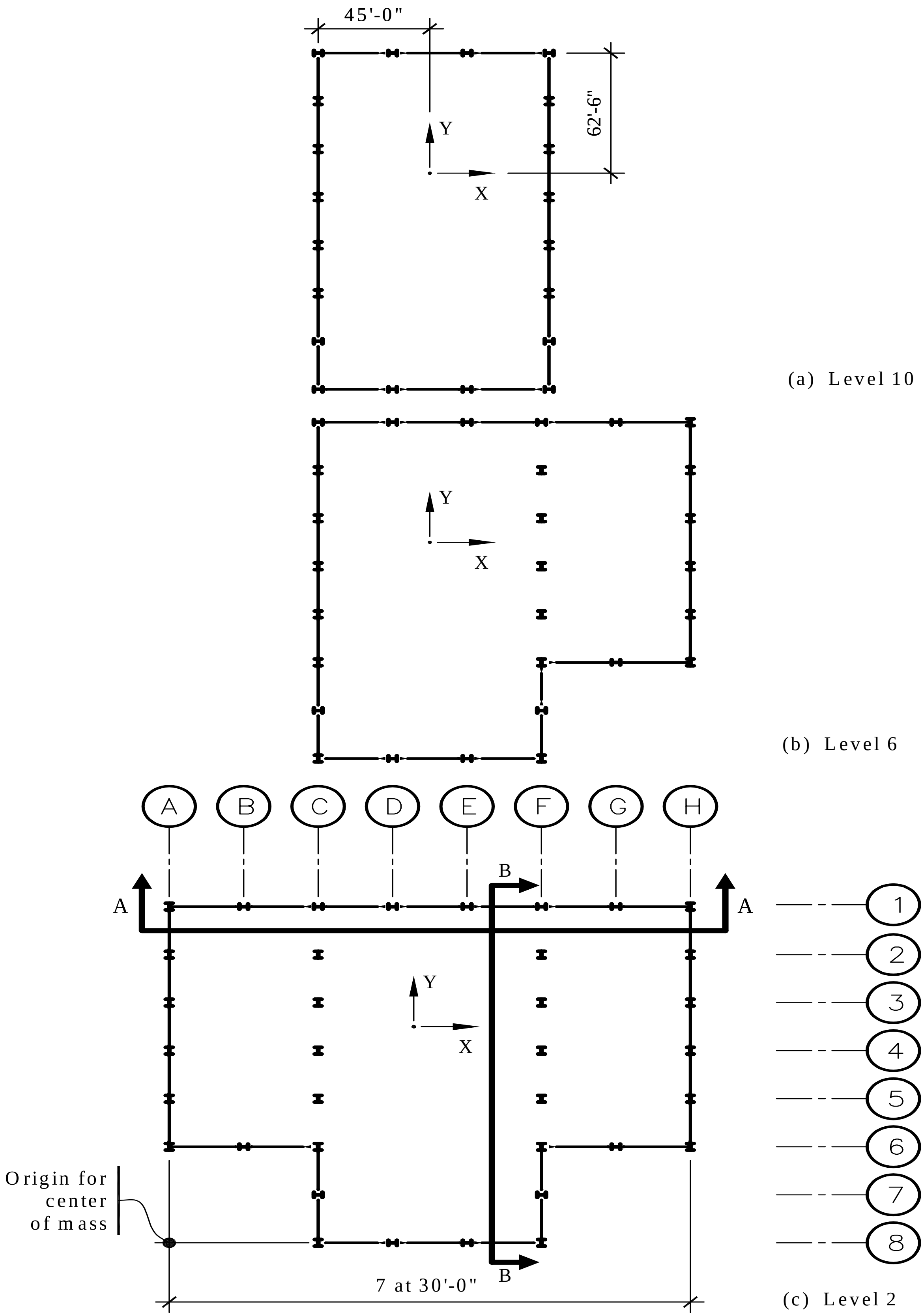
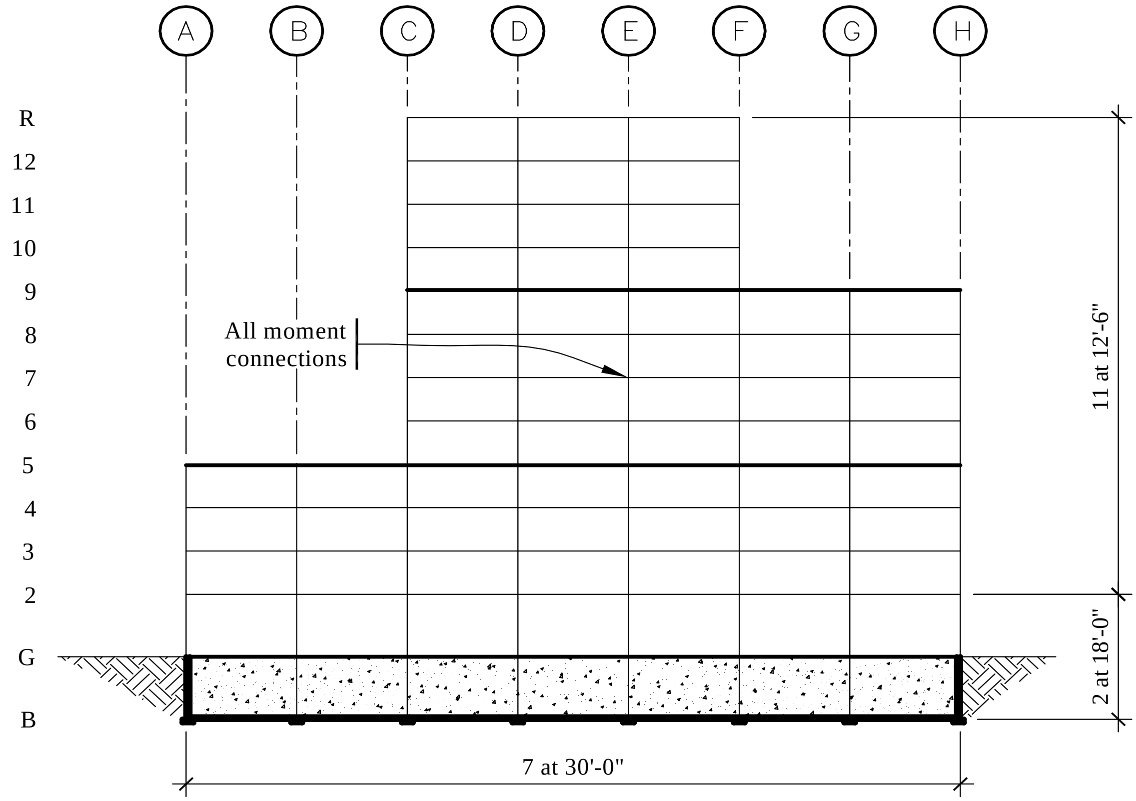
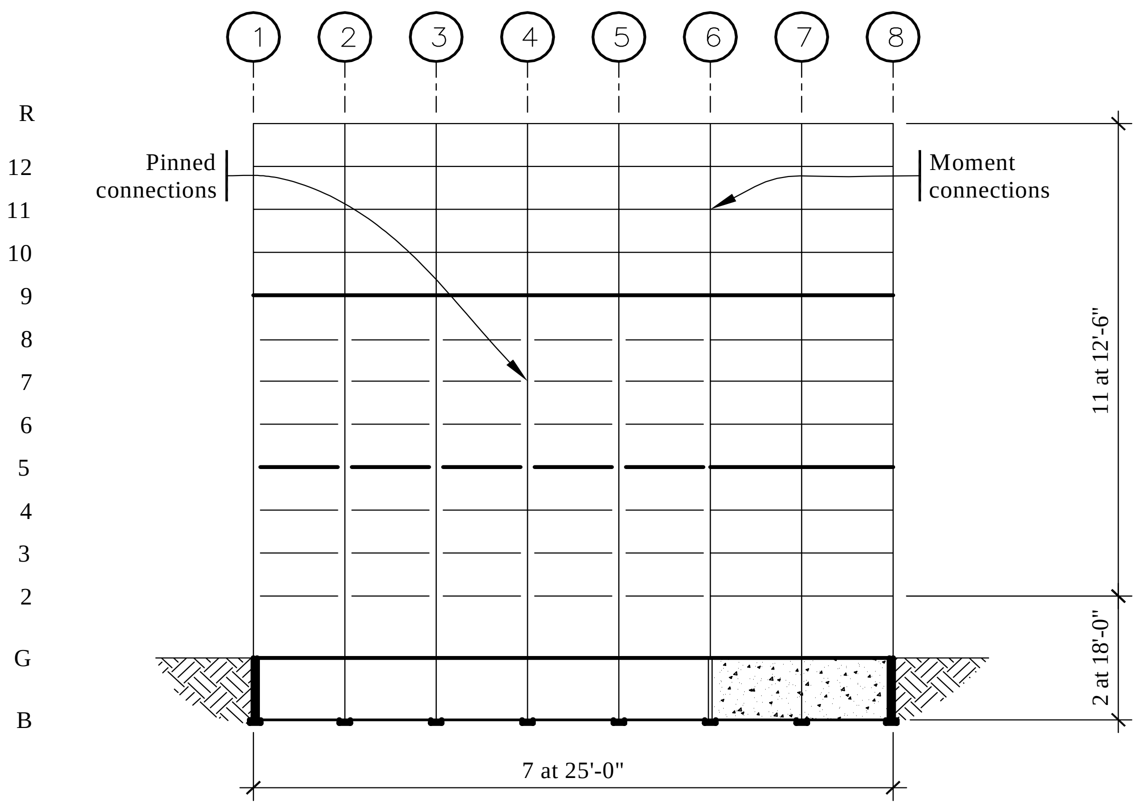


Figure 3.1-1 Various floor plans of 12-story Stockton building (1.0 ft = 0.3048 m).



Section A-A



Section B-B

Figure 3.1-2 Sections through Stockton building (1.0 ft. = 0.3048 m).

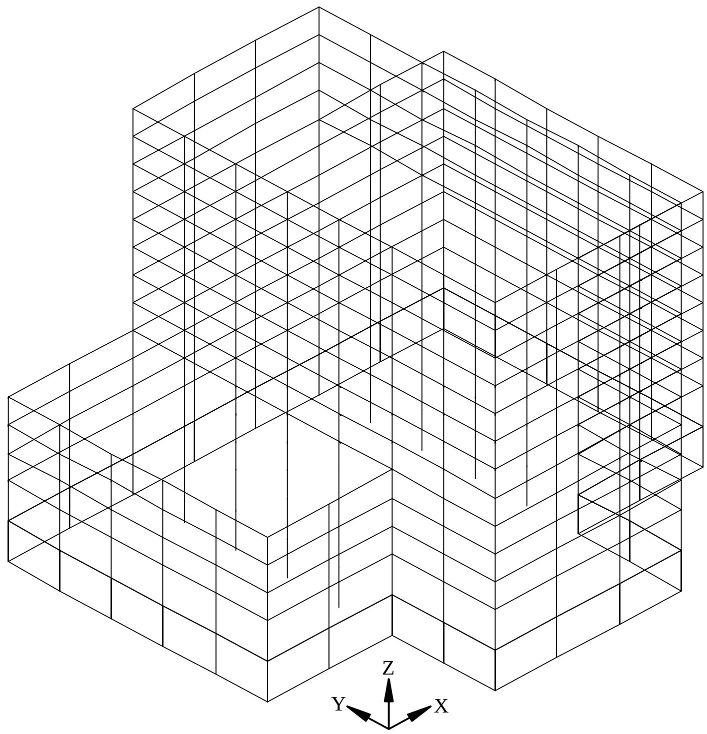


Figure 3.1-3 Three-dimensional wire-frame model of Stockton building.

The lateral-load-resisting system consists of special moment frames at the perimeter of the building and along Grids C and F. For the frames on Grids C and F, the columns extend down to the foundation, but the lateral-load-resisting girders terminate at Level 5 for Grid C and Level 9 for Grid F. Girders below these levels are simply connected. Due to the fact that the moment-resisting girders terminate in Frames C and F, much of the Y-direction seismic shears below Level 9 are transferred through the diaphragms to the frames on Grids A and H. Overturning moments developed in the upper levels of these frames are transferred down to the foundation by outriggering action provided by the columns. Columns in the moment-resisting frame range in size from W24x146 at the roof to W24x229 at Level G. Girders in the moment frames vary from W30x108 at the roof to W30x132 at Level G. Members of the moment resisting frames have a yield strength of 36 ksi, and floor members and interior columns that are sized strictly for gravity forces are 50 ksi.

3.1.3 *Provisions* Analysis Parameters

Stockton, California, is in San Joaquin County approximately 60 miles east of Oakland. According to *Provisions* Maps 7 and 8, the short-period and 1-second mapped spectral acceleration parameters are:

$S_s = 1.25$
$S_1 = 0.40$

[The 2003 *Provisions* have adopted the 2002 USGS probabilistic seismic hazard maps, and the maps have been added to the body of the 2003 *Provisions* as figures in Chapter 3 (instead of being issued in a separate map package).]

Assuming Site Class C, the adjusted maximum considered 5-percent-damped spectral accelerations are obtained from *Provisions* Eq. 4.1.2.4-1 and Eq. 4.1.2.4-2 [3.3-1 and 3.3-2]:

$$S_{MS} = F_a S_S = 1.0(1.25) = 1.25$$
$$S_{M1} = F_v S_1 = 1.4(0.4) = 0.56$$

where the coefficients F_a = 1.0 and F_v = 1.4 come from *Provisions* Tables 4.1.2.4(a) and 4.1.2.4(b) [3.3-1 and 3.3-2], respectively.

According to *Provisions* Eq. 4.1.2.5-1 and 4.1.2.5-2 [3.3-3 and 3.3-4], the design level spectral acceleration parameters are 2/3 of the above values:

$$S_{DS} = \frac{2}{3} S_{MS} = \frac{2}{3}(1.25) = 0.833$$

$$S_{D1} = \frac{2}{3} S_{M1} = \frac{2}{3}(0.56) = 0.373$$

As the primary occupancy of the building is business offices, the Seismic Use Group (SUG) is I and, according to *Provisions* Table 1.4 [1.3-1], the importance factor (*I*) is 1. According to *Provisions* Tables 4.2.1(a) and 4.2.1(b) [1.4-1 and 1.4-2], the Seismic Design Category (SDC) for this building is D.

The lateral-load-resisting system of the building is a special moment-resisting frame of structural steel. For this type of system, *Provisions* Table 5.2.2 [4.3-1] gives a response modification coefficient (*R*) of 8 and a deflection amplification coefficient (C_d) of 5.5. Note that there is no height limit placed on special moment frames.

According to *Provisions* Table 5.2.5.1 [4.4-1] if the building has certain types of irregularities or if the computed building period exceeds 3.5 seconds where $T_S = S_{D1}/S_{DS}$ = 0.45 seconds, the minimum level of analysis required for this structure is modal-response-spectrum analysis. This requirement is based on apparent plan and vertical irregularities as described in *Provisions* Tables 5.2.3.2 and 5.2.3.3 [4.3-2 and 4.3-3]. The ELF procedure would not be allowed for a final design but, as explained later, certain aspects of an ELF analysis are needed in the modal-response-spectrum analysis. For this reason, and for comparison purposes, a complete ELF analysis is carried out and described herein.

3.1.4 Dynamic Properties

Before any analysis can be carried out, it is necessary to determine the dynamic properties of the structure. These properties include mass, periods of vibration and their associated mode shapes, and damping.

3.1.4.1 Mass

For two-dimensional analysis, only the translational mass is required. To perform a three-dimensional modal or time-history analysis, it is necessary to compute the mass moment of inertia for floor plates rotating about the vertical axis and to find the location of the center of mass of each level of the structure. This may be done two different ways:

1. The mass moments of inertia may be computed "automatically" by SAP2000 by modeling the floor diaphragms as shell elements and entering the proper mass density of the elements. Line masses, such as window walls and exterior cladding, may be modeled as point masses. The floor diaphragms

may be modeled as rigid in-plane by imposing displacement constraints or as flexible in-plane by allowing the shell elements to deform in their own plane. Modeling the diaphragms as flexible is not necessary in most cases and may have the disadvantage of increasing solution time because of the additional number of degrees of freedom required to model the diaphragm.

2. The floor is assumed to be rigid in-plane but is modeled without explicit diaphragm elements. Displacement constraints are used to represent the in-plane rigidity of the diaphragm. In this case, floor masses are computed by hand (or an auxiliary program) and entered at the "master node" location of each floor diaphragm. The location of the master node should coincide with the center of mass of the floor plate. (Note that this is the approach traditionally used in programs such as ETABS which, by default, assumed rigid in-plane diaphragms and modeled the diaphragms using constraints.)

In the analysis performed herein, both approaches are illustrated. Final analysis used Approach 1, but the frequencies and mode shapes obtained from Approach 1 were verified with a separate model using Approach 2. The computation of the floor masses using Approach 2 is described below.

Due to the various sizes and shapes of the floor plates and to the different dead weights associated with areas within the same floor plate, the computation of mass properties is not easily carried out by hand. For this reason, a special purpose computer program was used. The basic input for the program consists of the shape of the floor plate, its mass density, and definitions of auxiliary masses such as line, rectangular, and concentrated mass.

The uniform area and line masses associated with the various floor plates are given in Tables 3.1-1 and 3.1-2. The line masses are based on a cladding weight of 15.0 psf, story heights of 12.5 or 18.0 ft, and parapets 4.0 ft high bordering each roof region. Figure 3.1-4 shows where each mass type occurs. The total computed floor mass, mass moment of inertia, and locations of center of mass are shown in Table 3.1-3. The reference point for center of mass location is the intersection of Grids A and 8. Note that the dimensional units of mass moment of inertia (in.-kip-sec^2/radian), when multiplied by angular acceleration (radians/sec^2), must yield units of torsional moment (in.-kips).

Table 3.1-3 includes a mass computed for Level G of the building. This mass is associated with an extremely stiff story (the basement level) and is not dynamically excited by the earthquake. As shown later, this mass is not included in equivalent lateral force computations.

Table 3.1-1 Area Masses on Floor Diaphragms

Mass Type	Area Mass Designation				
	A	B	C	D	E
Slab and Deck (psf)	50	75	50	75	75
Structure (psf)	20	20	20	20	50
Ceiling and Mechanical (psf)	15	15	15	15	15
Partition (psf)	10	10	0	0	10
Roofing (psf)	0	0	15	15	0
Special (psf)	0	0	0	60	25
TOTAL (psf)	95	120	100	185	175

See Figure 3.1-4 for mass location.
1.0 psf = 47.9 N/m^2.

Table 3.1-2 Line Masses on Floor Diaphragms

Mass Type	Line Mass Designation				
	1	2	3	4	5
From Story Above (plf)	60.0	93.8	93.8	93.8	135.0
From Story Below (plf)	93.8	93.8	0.0	135.0	1350.0
TOTAL (plf)	153.8	187.6	93.8	228.8	1485.0

See Figure 3.1-4 for mass location.
1.0 plf = 14.6 N/m.

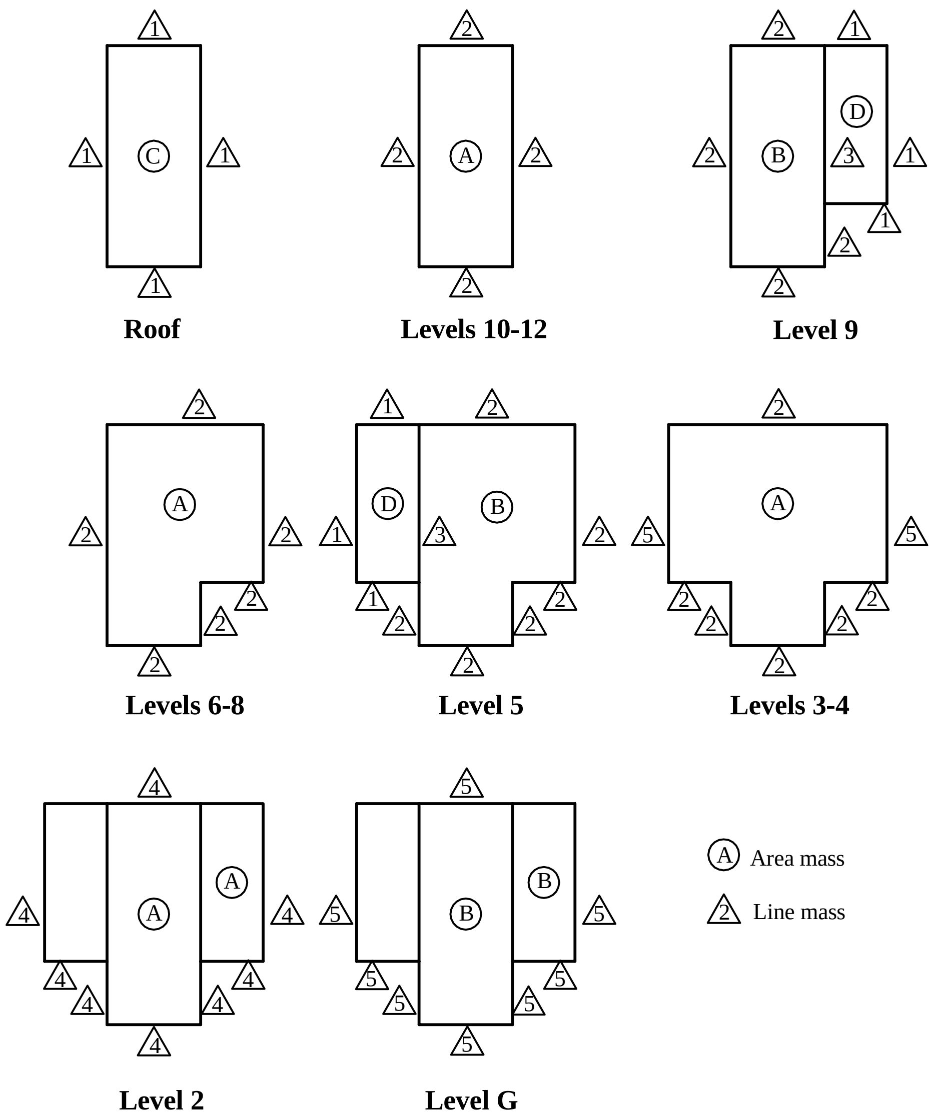


Figure 3.1-4 Key diagram for computation of floor mass.

Table 3.1-3 Floor Mass, Mass Moment of Inertia, and Center of Mass Locations

Level	Weight (kips)	Mass (kip-sec$^{2/}$in.)	Mass Moment of Inertia (in.-kip-sec$^{2/}$/radian)	X Distance to C.M. (in.)	Y Distance to C.M. (in.)
R	1656.5	4.287	2.072×10^6	1260	1050
12	1595.8	4.130	2.017×10^6	1260	1050
11	1595.8	4.130	2.017×10^6	1260	1050
10	1595.8	4.130	2.017×10^6	1260	1050
9	3403.0	8.807	5.309×10^6	1637	1175
8	2330.8	6.032	3.703×10^6	1551	1145
7	2330.8	6.032	3.703×10^6	1551	1145
6	2330.8	6.032	3.703×10^6	1551	1145
5	4323.8	11.190	9.091×10^6	1159	1212
4	3066.1	7.935	6.356×10^6	1260	1194
3	3066.1	7.935	6.356×10^6	1260	1194
2	3097.0	8.015	6.437×10^6	1260	1193
G	6526.3	16.890	1.503×10^7	1260	1187
Σ	36918.6				

1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.

3.1.4.2 Period of Vibration

3.1.4.2.1 Approximate Period of Vibration

The formula in *Provisions* Eq. 5.4.2.1-1 [5.2-6] is used to estimate the building period:

$$T_a = C_r h_n^x$$

where C_r = 0.028 and x = 0.8 for a steel moment frame from *Provisions* Table 5.4.2.1 [5.2-2]. Using h_n = the total building height (above grade) = 155.5 ft, $T_a = 0.028(155.5)^{0.8} = 1.59$ sec.

When the period is computed from a properly substantiated analysis, the *Provisions* requires that the computed period not exceed C_uT_a where C_u = 1.4 (from *Provisions* Table 5.4.2 [5.2-1] using S_{D1} = 0.373g). For the structure under consideration, C_uT_a = 1.4(1.59) = 2.23 seconds. When a modal-response spectrum is used, *Provisions* Sec. 5.5.7 [5.3.7] requires that the displacements, drift, and member design forces be scaled to a value consistent with 85 percent of the equivalent lateral force base shear computed using the period C_uT_a = 2.23 sec. *Provisions* Sec. 5.6.3 [5.4.3] requires that time-history analysis results be scaled up to an ELF shear consistent with $T = C_uT_a$ (without the 0.85 factor).[1]

Note that when the accurately computed period (such as from a Rayleigh analysis) is less than the approximate value shown above, the computed period should be used. In no case, however, must a period less than T_a = 1.59 seconds be used. The use of the Rayleigh method and the eigenvalue method of determining accurate periods of vibration are illustrated in a later part of this example.

[1]This requirements seems odd to the writer since the *Commentary* to the *Provisions* states that time-history analysis is superior to response-spectrum analysis. Nevertheless, the time-history analysis performed later will be scaled as required by the *Provisions*.

3.1.4.3 Damping

When a modal-response-spectrum analysis is performed, the structure's damping is included in the response spectrum. A damping ratio of 0.05 (5 percent of critical) is appropriate for steel structures. This is consistent with the level of damping assumed in the development of the mapped spectral acceleration values.

When recombining the individual modal responses, the square root of the sum of the squares (SRSS) technique has generally been replaced in practice by the complete quadratic combination (CQC) approach. Indeed, *Provisions* Sec. 5.5.7 [5.3.7] requires that the CQC approach be used when the modes are closely spaced. When using CQC, the analyst must correctly specify a damping factor. This factor must match that used in developing the response spectrum. It should be noted that if zero damping is used in CQC, the results are the same as those for SRSS.

For time-history analysis, SAP2000 allows an explicit damping ratio to be used in each mode. For this structure, a damping of 5 percent of critical was specified in each mode.

3.1.5 Equivalent Lateral Force Analysis

Prior to performing modal or time-history analysis, it is often necessary to perform an equivalent lateral force (ELF) analysis of the structure. This analysis typically is used for preliminary design and for assessing the three-dimensional response characteristics of the structure. ELF analysis is also useful for investigating the behavior of drift-controlled structures, particularly when a virtual force analysis is used for determining member displacement participation factors.[2] The virtual force techniques cannot be used for modal-response-spectrum analysis because signs are lost in the CQC combinations.

In anticipation of the "true" computed period of the building being greater than 2.23 seconds, the ELF analysis is based on a period of vibration equal to C_uT_a = 2.23 seconds. For the ELF analysis, it is assumed that the structure is "fixed" at grade level. Hence, the total effective weight of the structure (see Table 3.1-3) is the total weight minus the grade level weight, or 36918.6 - 6526.3 = 30392.3 kips.

3.1.5.1 Base Shear and Vertical Distribution of Force

Using *Provisions* Eq. 5.4.1 [5.2-1], the total seismic shear is:

$$V = C_S W$$

where W is the total weight of the structure. From *Provisions* Eq. 5.4.1.1-1 [5.2-2], the maximum (constant acceleration region) spectral acceleration is:

$$C_{S_{max}} = \frac{S_{DS}}{(R/I)} = \frac{0.833}{(8/1)} = 0.104$$

[2]For an explanation of the use of the virtual force technique, see "Economy of Steel Framed Structures Through Identification of Structural Behavior" by F. Charney, *Proceedings of the 1993 AISC Steel Construction Conference*, Orlando, Florida, 1993.

Provisions Eq. 5.4.1.1-2 [5.2-3] controls in the constant velocity region:

$$C_S = \frac{S_{D1}}{T(R/I)} = \frac{0.373}{2.23(8/1)} = 0.021$$

However, the acceleration must not be less than that given by *Provisions* Eq. 5.4.1.1-3 [replaced by 0.010 in the 2003 *Provisions*]:

$$C_{S_{min}} = 0.044 I S_{DS} = 0.044(1)(0.833) = 0.037$$

[With the change of this base shear equation, the result of Eq. 5.2-3 would control, reducing the design base shear significantly. This change would also result in removal of the horizontal line in Figure 3.1-5 and the corresponding segment of Figure 3.1-6.]

The value from Eq. 5.4.1.1-3 [not applicable in the 2003 *Provisions*] controls for this building. Using W = 30,392 kips, V = 0.037(30,392) = 1,124 kips. The acceleration response spectrum given by the above equations is plotted in Figure 3.1-5.

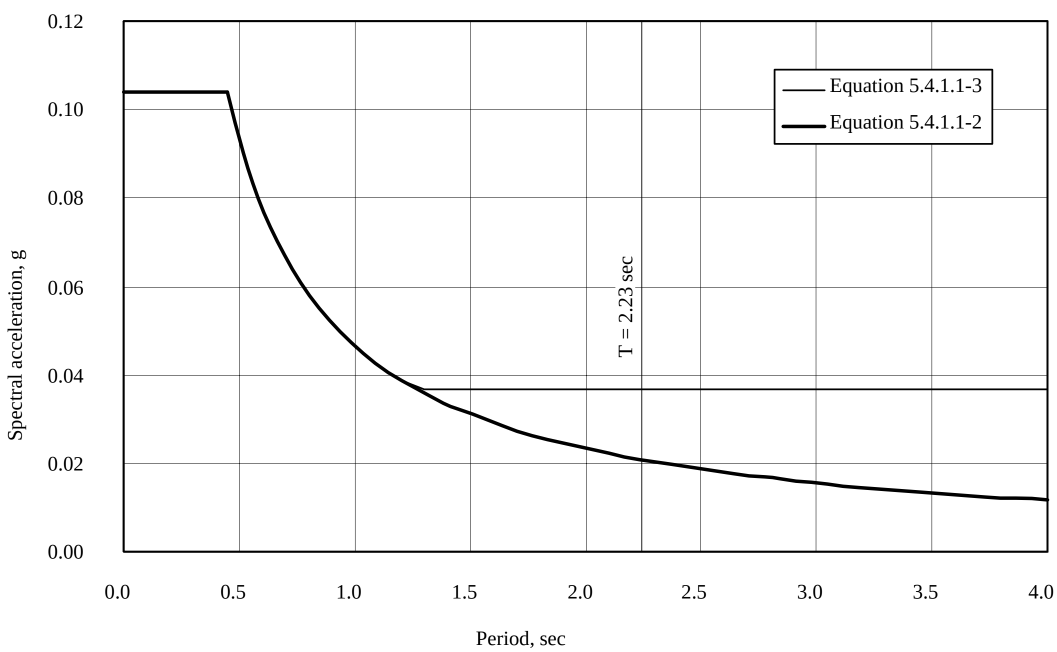


Figure 3.1-5 Computed ELF total acceleration response spectrum.

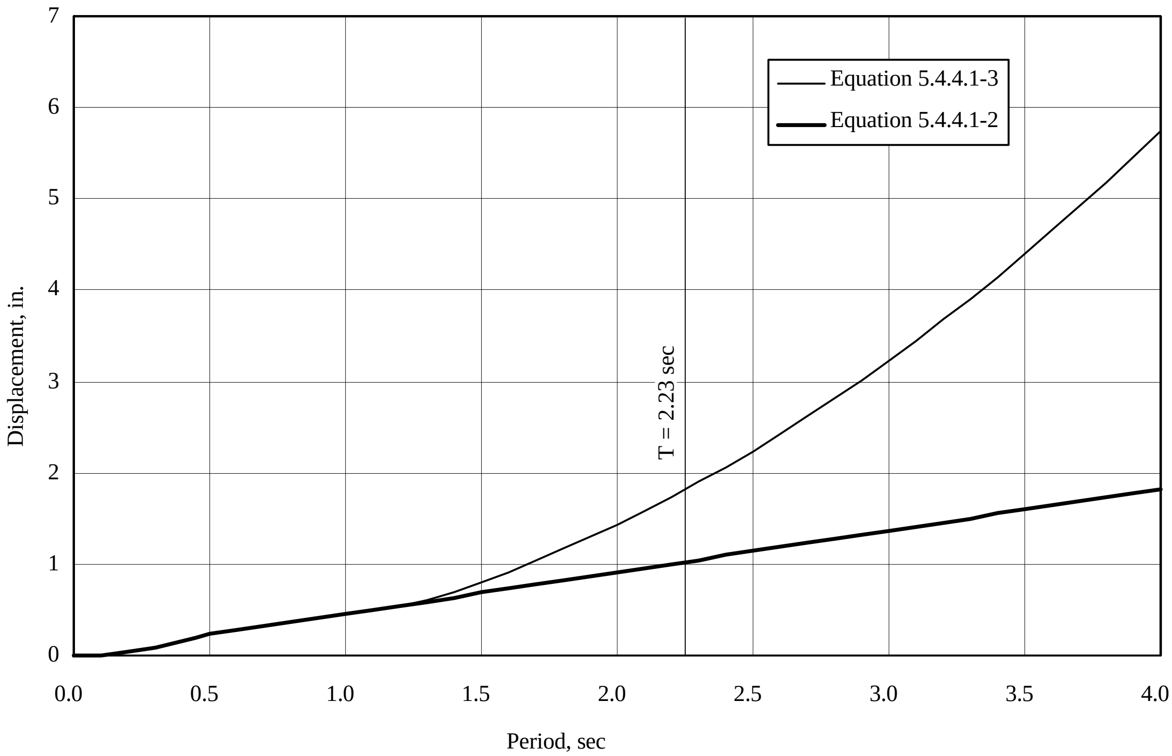


Figure 3.1-6 Computed ELF relative displacement response spectrum (1.0 in. = 25.4 mm).

While it is certainly reasonable to enforce a minimum base shear, *Provisions* Sec. 5.4.6.1 has correctly recognized that displacements predicted using Eq. 5.4.1.1-3 are not reasonable. Therefore, it is very important to note that *Provisions* Eq. 5.4.1.1-3, when it controls, should be used for determining member forces, but should not be used for computing drift. For drift calculations, forces computed according to Eq. 5.4.1.1-2 [5.2-3]should be used. The effect of using Eq. 5.4.1.1-3 for drift is shown in Figure 3.1-6, where it can be seen that the fine line, representing Eq. 5.4.1.1-3, will predict significantly larger displacements than Eq. 5.4.1.1-2 [5.2-3].

[The minimum base shear is 1% of the weight in the 2003 *Provisions* (C_S = 0.01). For this combination of S_{D1} and R, the new minimum controls for periods larger than 4.66 second. The minimum base shear equation for near-source sites (now triggered in the *Provisions* by S_1 greater than or equal to 0.6) has been retained.]

In this example, all ELF analysis is performed using the forces obtained from Eq. 5.4.1.1-3, but for the purposes of computing drift, the story deflections computed using the forces from Eq. 5.4.1.1-3 are multiplied by the ratio (0.021/0.037 = 0.568).

The base shear computed according to *Provisions* Eq. 5.4.1.1-3 is distributed along the height of the building using *Provisions* Eq. 5.4.3.1 and 5.4.3.2 [5.2-10 and 5.2-11]:

$$F_x = C_{vx} V$$

and

$$C_{vx} = \frac{w_x h^k}{\sum_{i=1}^{n} w_i h_i^k}$$

where k = 0.75 + 0.5T = 0.75 + 0.5(2.23) = 1.86. The story forces, story shears, and story overturning moments are summarized in Table 3.1-4.

Table 3.1-4 Equivalent Lateral Forces for Building Responding in X and Y Directions

Level x	w_x (kips)	h_x (ft)	$w_x h_x^k$	C_{vx}	F_x (kips)	V_x (kips)	M_x (ft-kips)
R	1656.5	155.5	20266027	0.1662	186.9	186.9	2336
12	1595.8	143.0	16698604	0.1370	154.0	340.9	6597
11	1595.8	130.5	14079657	0.1155	129.9	470.8	12482
10	1595.8	118.0	11669128	0.0957	107.6	578.4	19712
9	3403.0	105.5	20194253	0.1656	186.3	764.7	29271
8	2330.8	93.0	10932657	0.0897	100.8	865.5	40090
7	2330.8	80.5	8352458	0.0685	77.0	942.5	51871
6	2330.8	68.0	6097272	0.0500	56.2	998.8	64356
5	4323.8	55.5	7744119	0.0635	71.4	1070.2	77733
4	3066.1	43.0	3411968	0.0280	31.5	1101.7	91505
3	3066.1	30.5	1798066	0.0147	16.6	1118.2	103372
2	3097.0	18.0	679242	0.0056	6.3	1124.5	120694
Σ	30392.3	-	121923430	1.00	1124.5		

1.0 ft = 0.3048 m, 1.0 kip = 4.45 kN.

3.1.5.2 Accidental Torsion and Orthogonal Loading Effects

When using the ELF method as the basis for structural design, two effects must be added to the direct lateral forces shown in Table 3.1-4. The first of these effects accounts for the fact that the earthquake can produce inertial forces that act in any direction. For SDC D, E, and F buildings, *Provisions* Sec. 5.2.5.2.3 [4.4.2.3] requires that the structure be investigated for forces that act in the direction that causes the "critical load effect." Since this direction is not easily defined, the *Provisions* allows the analyst to load the structure with 100 percent of the seismic force in one direction (along the X axis, for example) simultaneous with the application of 30 percent of the force acting in the orthogonal direction (the Y axis).

The other requirement is that the structure be modeled with additional forces to account for uncertainties in the location of center of mass and center of rigidity, uneven yielding of vertical systems, and the possibility of torsional components of ground motion. This requirement, given in *Provisions* Sec. 5.4.4.2 [5.2.4.2], can be satisfied for torsionally regular buildings by applying the equivalent lateral force at an eccentricity, where the eccentricity is equal to 5 percent of the overall dimension of the structure in the direction perpendicular to the line of the application of force.

For structures in SDC C, D, E, or F, these accidental eccentricities (and inherent torsion) must be amplified if the structure is classified as torsionally irregular. According to *Provisions* Table 5.2.3.2, a torsional irregularity exists if:

$$\frac{\delta_{max}}{\delta_{avg}} \geq 1.2$$

where, as shown in Figure 3.1-7, δ_{max} is the maximum displacement at the edge of the floor diaphragm, and δ_{avg} is the average displacement of the diaphragm. If the ratio of displacements is greater than 1.4, the torsional irregularity is referred to as "extreme." In computing the displacements, the structure must be loaded with the basic equivalent lateral forces applied at a 5 percent eccentricity.

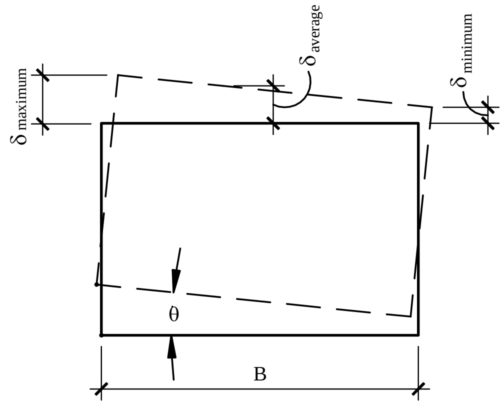


Figure 3.1-7 Amplification of accidental torsion.

The analysis of the structure for accidental torsion was performed on SAP2000. The same model was used for ELF, modal-response-spectrum, and modal-time-history analysis. The following approach was used for the mathematical model of the structure:

1. The floor diaphragm was modeled as infinitely rigid in-plane and infinitely flexible out-of-plane. Shell elements were used to represent the diaphragm mass. Additional point masses were used to represent cladding and other concentrated masses.

2. Flexural, shear, axial, and torsional deformations were included in all columns. Flexural, shear, and torsional deformations were included in the beams. Due to the rigid diaphragm assumption, axial deformation in beams was neglected.

3. Beam-column joints were modeled using centerline dimensions. This approximately accounts for deformations in the panel zone.

4. Section properties for the girders were based on bare steel, ignoring composite action. This is a reasonable assumption in light of the fact that most of the girders are on the perimeter of the building and are under reverse curvature.

5. Except for those lateral-load-resisting columns that terminate at Levels 5 and 9, all columns were assumed to be fixed at their base.

The results of the accidental torsion analysis are shown in Tables 3.1-5 and 3.1-6. As may be observed, the largest ratio of maximum to average floor displacements is 1.16 at Level 5 of the building under Y direction loading. Hence, this structure is not torsionally irregular and the story torsions do not need to be amplified.

Table 3.1-5 Computation for Torsional Irregularity with ELF Loads Acting in X Direction

Level	δ_1 (in.)	δ_2 (in.)	δ_{avg} (in.)	δ_{max} (in.)	$\delta_{max}/\delta_{avg}$	Irregularity
R	6.04	7.43	6.74	7.43	1.10	none
12	5.75	7.10	6.43	7.10	1.11	none
11	5.33	6.61	5.97	6.61	1.11	none
10	4.82	6.01	5.42	6.01	1.11	none
9	4.26	5.34	4.80	5.34	1.11	none
8	3.74	4.67	4.21	4.67	1.11	none
7	3.17	3.96	3.57	3.96	1.11	none
6	2.60	3.23	2.92	3.23	1.11	none
5	2.04	2.52	2.28	2.52	1.11	none
4	1.56	1.91	1.74	1.91	1.10	none
3	1.07	1.30	1.19	1.30	1.10	none
2	0.59	0.71	0.65	0.71	1.09	none

Tabulated displacements are not amplified by C_d. Analysis includes accidental torsion. 1.0 in. = 25.4 mm.

Table 3.1-6 Computation for Torsional Irregularity with ELF Loads Acting in Y Direction

Level	δ_1 (in.)	δ_2 (in.)	δ_{avg} (in.)	δ_{max} (in)	$\delta_{max}/\delta_{avg}$	Irregularity
R	5.88	5.96	5.92	5.96	1.01	none
12	5.68	5.73	5.71	5.73	1.00	none
11	5.34	5.35	5.35	5.35	1.00	none
10	4.92	4.87	4.90	4.92	1.01	none
9	4.39	4.29	4.34	4.39	1.01	none
8	3.83	3.88	3.86	3.88	1.01	none
7	3.19	3.40	3.30	3.40	1.03	none
6	2.54	2.91	2.73	2.91	1.07	none
5	1.72	2.83	2.05	2.38	1.16	none
4	1.34	1.83	1.59	1.83	1.15	none
3	0.93	1.27	1.10	1.27	1.15	none
2	0.52	0.71	0.62	0.71	1.15	none

Tabulated displacements are not amplified by C_d. Analysis includes accidental torsion. 1.0 in. = 25.4 mm.

3.1.5.3 Drift and P-Delta Effects

Using the basic structural configuration shown in Figure 3.1-1 and the equivalent lateral forces shown in Table 3.1-4, the total story deflections were computed as shown in the previous section. In this section, story drifts are computed and compared to the allowable drifts specified by the *Provisions*.

The results of the analysis are shown in Tables 3.1-7 and 3.1-8. The tabulated drift values are somewhat different from those shown in Table 3.1-5 because the analysis for drift did not include accidental torsion, whereas the analysis for torsional irregularity did. In Tables 3.1-7 and 3.1-8, the values in the first numbered column are the *average* story displacements computed by the SAP2000 program using the lateral forces of Table 3.1-4. Average story drifts are used here instead of maximum story drifts because this structure does not have a "significant torsional response." If the torsional effect were significant, the maximum drifts at the extreme edge of the diaphragm would need to be checked.

The values in column 2 of Tables 3.1-7 and 3.1-8 are the story drifts as reported by SAP2000. These drift values, however, are much less than those that will actually occur because the structure will respond inelastically to the earthquake. The true inelastic story drift, which by assumption is equal to C_d = 5.5

times the SAP2000 drift, is shown in Column 3. As discussed above in Sec. 3.1.5.1, the values in column 4 are multiplied by 0.568 to scale the results to the base shear calculated ignoring *Provisions* Eq. 5.4.1.1-3 since that limit does not apply to drift checks. [Recall that the minimum base shear is different in the 2003 *Provisions*.] The allowable story drift of 2.0 percent of the story height per *Provisions* Table 5.2-8 is shown in column 5. (Recall that this building is assigned to Seismic Use Group I.) It is clear from Tables 3.1-7 and 3.1-8 that the allowable drift is not exceeded at any level.

Table 3.1-7 ELF Drift for Building Responding in X Direction

Level	1 Total Drift from SAP2000 (in.)	2 Story Drift from SAP2000 (in.)	3 Inelastic Story Drift (in.)	4 Inelastic Drift Times 0.568 (in.)	5 Allowable Drift (in.)
R	6.71	0.32	1.73	0.982	3.00
12	6.40	0.45	2.48	1.41	3.00
11	5.95	0.56	3.08	1.75	3.00
10	5.39	5.39	3.38	1.92	3.00
9	4.77	0.59	3.22	1.83	3.00
8	4.19	0.64	3.52	2.00	3.00
7	3.55	0.65	3.58	2.03	3.00
6	2.90	0.63	3.44	1.95	3.00
5	2.27	0.55	3.00	1.70	3.00
4	1.73	0.55	3.00	1.70	3.00
3	1.18	0.54	2.94	1.67	3.00
2	0.65	0.65	3.55	2.02	4.32

Column 4 adjusts for *Provisions* Eq. 5.4.1.1-2 (for drift) vs 5.4.1.1-3 (for strength). [Such a modification is not necessary when using the 2003 *Provisions* because the minimum base shear is different. Instead, the design forces applied to the model, which produce the drifts in Columns 1 and 2, would be lower by a factor of 0.568.]
1.0 in. = 25.4 mm.

Table 3.1-8 ELF Drift for Building Responding in Y Direction

Level	1 Total Drift from SAP2000 (in.)	2 Story Drift from SAP2000 (in.)	3 Inelastic Story Drift (in.)	4 Inelastic Drift Times 0.568 (in.)	5 Allowable Drift (in.)
R	6.01	0.22	1.21	0.687	3.00
12	5.79	0.36	1.98	1.12	3.00
11	5.43	0.45	2.48	1.41	3.00
10	4.98	0.67	3.66	2.08	3.00
9	4.32	0.49	2.70	1.53	3.00
8	3.83	0.57	3.11	1.77	3.00
7	3.26	0.58	3.19	1.81	3.00
6	2.68	0.64	3.49	1.98	3.00
5	2.05	0.46	2.53	1.43	3.00
4	1.59	0.49	2.67	1.52	3.00
3	1.10	0.49	2.70	1.53	3.00
2	0.61	0.61	3.36	1.91	4.32

Column 4 adjusts for *Provisions* Eq. 5.4.1.1-2 (for drift) vs 5.4.1.1-3 (for strength). [Such a modification is not necessary when using the 2003 *Provisions* because the minimum base shear is different. Instead, the design forces applied to the model, which produce the drifts in Columns 1 and 2, would be lower by a factor of 0.568.]
1.0 in. = 25.4 mm.

3.1.5.3.1 Using ELF Forces and Drift to Compute Accurate Period

Before continuing with the example, it is helpful to use the computed drifts to more accurately estimate the fundamental periods of vibration of the building. This will serve as a check on the “exact” periods computed by eigenvalue extraction in SAP2000. A Rayleigh analysis will be used to estimate the periods. This procedure, which is usually very accurate, is derived as follows:

The exact frequency of vibration ω (a scalar), in units of radians/second, is found from the following eigenvalue equation:

$$K\phi = \omega^2 M\phi$$

where K is the structure stiffness matrix, M is the (diagonal) mass matrix, and ϕ, is a vector containing the components of the mode shape associated with ω.

If an approximate mode shape δ is used instead of ϕ, where δ is the deflected shape under the equivalent lateral forces F, the frequency ω can be closely approximated. Making the substitution of δ for ϕ, premultiplying both sides of the above equation by δ^T (the transpose of the displacement vector), noting that $F = K\delta$, and $M = (1/g)W$, the following is obtained:

$$\delta^T F = \omega^2 \delta^T M \delta = \frac{\omega^2}{g}\delta^T W \delta$$

where W is a vector containing the story weights and g is the acceleration due to gravity (a scalar). After rearranging terms, this gives:

$$\omega = \sqrt{g\frac{\delta^T F}{\delta^T W \delta}}$$

Using the relationship between period and frequency, $T = \frac{2\pi}{\omega}$.

Using F from Table 3.1-4 and δ from Column 1 of Tables 3.1-7 and 3.1-8, the periods of vibration are computed as shown in Tables 3.1-9 and 3.1-10 for the structure loaded in the X and Y directions, respectively. As may be seen from the tables, the X-direction period of 2.87 seconds and the Y-direction period of 2.73 seconds are much greater than the approximate period of T_a = 1.59 seconds and also exceed the upper limit on period of C_uT_a = 2.23 seconds.

Table 3.1-9 Rayleigh Analysis for X-Direction Period of Vibration

Level	Drift, δ (in.)	Force, *F* (kips)	Weight, *W* (kips)	δ*F* (in.-kips)	$\delta^2 W/g$ (in.-kips-sec^2)
R	6.71	186.9	1656	1259.71	194.69
12	6.40	154.0	1598	990.22	170.99
11	5.95	129.9	1598	775.50	147.40
10	5.39	107.6	1598	583.19	121.49
9	4.77	186.3	3403	894.24	202.91
8	4.19	100.8	2330	424.37	106.88
7	3.55	77.0	2330	274.89	76.85
6	2.90	56.2	2330	164.10	51.41
5	2.27	71.4	4323	162.79	58.16
4	1.73	31.5	3066	54.81	24.02
3	1.18	16.6	3066	19.75	11.24
2	0.65	6.3	3097	4.10	3.39
Σ				5607.64	1169.42

$\omega = (5607/1169)^{0.5}$ = 2.19 rad/sec. $T = 2\pi/\omega$ = 2.87 sec. 1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.

Table 3.1-10 Rayleigh Analysis for Y-Direction Period of Vibration

Level	Drift, δ (in.)	Force, F (kips)	Weight, W (kips)	δF	$\delta^2 W/g$
R	6.01	186.9	1656	1123.27	154.80
12	5.79	154.0	1598	891.66	138.64
11	5.43	129.9	1598	705.36	121.94
10	4.98	107.6	1598	535.85	102.56
9	4.32	186.3	3403	804.82	164.36
8	3.83	100.8	2330	386.06	88.45
7	3.26	77.0	2330	251.02	64.08
6	2.68	56.2	2330	150.62	43.31
5	2.05	71.4	4323	146.37	47.02
4	1.59	31.5	3066	50.09	20.06
3	1.10	16.6	3066	18.26	9.60
2	0.61	6.3	3097	3.84	2.98
Σ				5067.21	957.81

$\omega = (5067/9589)^{0.5} = 2.30$ rad/sec. $T = 2\pi/\omega = 2.73$ sec. 1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.

3.1.5.3.2 P-Delta Effects

P-delta effects are computed for the X-direction response in Table 3.1-11. The last column of the table shows the story stability ratio computed according to *Provisions* Eq. 5.4.6.2-1 [5.2-16]:

$$\theta = \frac{P_x \Delta}{V_x h_{sx} C_d}$$

[In the 2003 *Provisions*, the equation for the story stability ratio was changed by introducing the importance factor (I) to the numerator. As previously formulated, larger axial loads (P_x) would be permitted where the design shears (V_x) included an importance factor greater than 1.0; that effect was unintended.]

Provisions Eq. 5.4.6.2-2 places an upper limit on θ:

$$\theta_{max} = \frac{0.5}{\beta C_d}$$

where β is the ratio of shear demand to shear capacity for the story. Conservatively taking $\beta = 1.0$ and using $C_d = 5.5$, $\theta_{max} = 0.091$. [In the 2003 *Provisions*, this upper limit equation has been eliminated. Instead, the *Provisions* require that where $\theta > 0.10$ a special analysis be performed in accordance with Sec. A5.2.3. This example constitutes a borderline case as the maximum stability ratio (at Level 3, as shown in Table 3.1-11) is 0.103.]

The Δ terms in Table 3.1-11 below are taken from Column 3 of Table 3.1-7 because these are consistent with the ELF story shears of Table 3.1-4 and thereby represent the true lateral stiffness of the system. (If 0.568 times the story drifts were used, then 0.568 times the story shears also would need to be used. Hence, the 0.568 factor would cancel out as it would appear in both the numerator and denominator.)

Table 3.1-11 Computation of P-Delta Effects for X-Direction Response

Level	h_{sx} (in.)	Δ (in.)	P_D (kips)	P_L (kips)	P_T (kips)	P_X (kips)	V_X(kips)	θ_X
R	150	1.73	1656.5	315.0	1971.5	1971.5	186.9	0.022
12	150	2.48	1595.8	315.0	1910.8	3882.3	340.9	0.034
11	150	3.08	1595.8	315.0	1910.8	5793.1	470.8	0.046
10	150	3.38	1595.8	315.0	1910.8	7703.9	578.4	0.055
9	150	3.22	3403.0	465.0	3868.0	11571.9	764.7	0.059
8	150	3.52	2330.8	465.0	2795.8	14367.7	865.8	0.071
7	150	3.58	2330.8	465.0	2795.8	17163.5	942.5	0.079
6	150	3.44	2330.8	465.0	2795.8	19959.3	998.8	0.083
5	150	3.00	4323.8	615.0	4938.8	24898.1	1070.2	0.085
4	150	3.00	3066.1	615.0	3681.1	28579.2	1101.7	0.094
3	150	2.94	3066.1	615.0	3681.1	32260.3	1118.2	0.103
2	216	3.55	3097.0	615.0	3712.0	35972.3	1124.5	0.096

1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.

The gravity force terms include a 20 psf uniform live load over 100 percent of the floor and roof area. The stability ratio just exceeds 0.091 at Levels 2 through 4. However, β was very conservatively taken as 1.0. Because a more refined analysis would most likely show a lower value of β, we will proceed assuming that P-delta effects are not a problem for this structure. Calculations for the Y direction produced similar results, but are not included herein.

3.1.5.4 Computation of Member Forces

Before member forces may be computed, the proper load cases and combinations of load must be identified such that all critical seismic effects are captured in the analysis.

3.1.5.4.1 Orthogonal Loading Effects and Accidental Torsion

For a nonsymmetric structure such as the one being analyzed, four directions of seismic force (+X, -X, +Y, -Y) must be considered and, for each direction of force, there are two possible directions for which the accidental eccentricity can apply (causing positive or negative torsion). This requires a total of eight possible combinations of direct force plus accidental torsion. When the 30 percent orthogonal loading rule is applied, the number of load combinations increases to 16 because, for each direct application of load, a positive or negative orthogonal loading can exist. Orthogonal loads are applied without accidental eccentricity.

Figure 3.1-8 illustrates the basic possibilities of application of load. Although this figure shows 16 different load combinations, it may be observed that eight of these combinations – 7, 8, 5, 6, 15, 16, 13, and 14 – are negatives of one of Combinations 1, 2, 3, 4, 9, 10, 11, and 12, respectively.

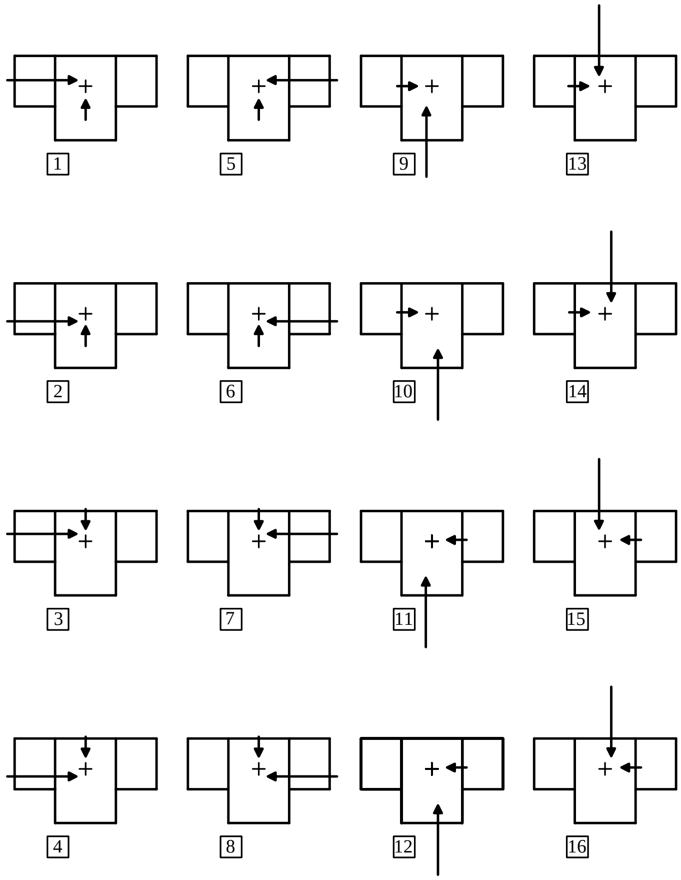


Figure 3.1-8 Basic load cases used in ELF analysis.

3.1.5.4.2 Load Combinations

The basic load combinations for this structure come from ASCE 7 with the earthquake loadings modified according to *Provisions* Sec. 5.2.7 [4.2.2.1].

The basic ASCE 7 load conditions that include earthquake are:

$$1.4D + 1.2L + E + 0.2S$$

and

$$0.9D + E$$

From *Provisions* Eq. 5.2.7-1 and Eq. 5.2.7-2 [4.2-1 and 4.2-2]:

$$E = \rho Q_E + 0.2S_{DS}Q_D$$

and

$$E = \rho Q_E - 0.2S_{DS}Q_D$$

where ρ is a redundancy factor (explained later), Q_E is the earthquake load effect, Q_D is the dead load effect, and S_{DS} is the short period spectral design acceleration.

Using S_{DS} = 0.833 and assuming the snow load is negligible in Stockton, California, the basic load combinations become:

$$1.37D + 0.5L + \rho E$$

and

$$0.73D + \rho E$$

[The redundancy requirements have been changed substantially in the 2003 *Provisions*. Instead of performing the calculations that follow, 2003 *Provisions* Sec. 4.3.3.2 would require that an analysis determine the most severe effect on story strength and torsional response of loss of moment resistance at the beam-to-column connections at both ends of any single beam. Where the calculated effects fall within permitted limits, or the system is configured so as to satisfy prescriptive requirements in the exception, the redundancy factor is 1.0. Otherwise, ρ = 1.3. Although consideration of all possible single beam failures would require substantial effort, in most cases an experienced analyst would be able to identify a few critical elements that would be likely to produce the maximum effects and then explicitly consider only those conditions.]

Based on *Provisions* Eq. 5.2.4.2, the redundancy factor (ρ) is the largest value of ρ_x computed for each story:

$$\rho_x = 2 - \frac{20}{r_{max_x}\sqrt{A_x}}$$

In this equation, r_{max_x} is a ratio of element shear to story shear, and A_x is the area of the floor diaphragm immediately above the story under consideration; ρ_x need not be taken greater than 1.5, but it may not be less than 1.0. [In the 2003 *Provisions*, ρ is either 1.0 or 1.3.]

For this structure, the check is illustrated for the lower level only where the area of the diaphragm is 30,750 ft^2. Figure 3.1-1 shows that the structure has 18 columns resisting load in the X direction and 18 columns resisting load in the Y direction. If it is assumed that each of these columns equally resists base shear and the check, as specified by the *Provisions*, is made for any two adjacent columns:

$$r_{max_x} = 2/18 = 0.111 \text{ and } \rho_x = 2 - \frac{20}{0.11\sqrt{30750}} = 0.963 .$$

Checks for upper levels will produce an even lower value of ρ_x; therefore, ρ_x may be taken a 1.0 for this structure. Hence, the final load conditions to be used for design are:

$$1.37D + 0.5L + E$$

and

$$0.73D + E$$

The first load condition will produce the maximum negative moments (tension on the top) at the face of the supports in the girders and maximum compressive forces in columns. The second load condition will produce the maximum positive moments (or minimum negative moment) at the face of the supports of the girders and maximum tension (or minimum compression) in the columns. In addition to the above load condition, the gravity-only load combinations as specified in ASCE 7 also must be checked. Due to the relatively short spans in the moment frames, however, it is not expected that the non-seismic load combinations will control.

3.1.5.4.3 Setting up the Load Combinations in SAP2000

The load combinations required for the analysis are shown in Table 3.1-12.

It should be noted that 32 different load combinations are required only if one wants to maintain the signs in the member force output, thereby providing complete design envelopes for all members. As mentioned later, these signs are lost in response-spectrum analysis and, as a result, it is possible to capture the effects of dead load plus live load plus-or-minus earthquake load in a single SAP2000 run containing only four load combinations.

Table 3.1-12 Seismic and Gravity Load Combinations as Run on SAP 2000

Run	Combination	Lateral*		Gravity	
		A	B	1 (Dead)	2 (Live)
One	1	[1]		1.37	0.5
	2	[1]		0.73	0.0
	3	[-1]		1.37	0.5
	4	[-1]		0.73	0.0
	5		[2]	1.37	0.5
	6		[2]	0.73	0.0
	7		[-2]	1.37	0.5
	8		[-2]	0.73	0.0
Two	1	[3]		1. 37	0.5
	2	[3]		0.73	0.0
	3		[4]	1. 37	0.5
	4		[4]	0.73	0.0
	5	[-3]		1. 37	0.5
	6	[-3]		0.73	0.0
	7		[-4]	1. 37	0.5
	8		[-4]	0.73	0.0
Three	1	[9]		1. 37	0.5
	2	[9]		0.73	0.0
	3		[10]	1. 37	0.5
	4		[10]	0.73	0.0
	5	[-9]		1. 37	0.5
	6	[-9]		0.73	0.0
	7		[-10]	1. 37	0.5
	8		[-10]	0.73	0.0
Four	1	[11]		1. 37	0.5
	2	[11]		0.73	0.0
	3		[12]	1. 37	0.5
	4		[12]	0.73	0.0
	5	[-11]		1. 37	0.5
	6	[-11]		0.73	0.0
	7		[-12]	1. 37	0.5
	8		[-12]	0.73	0.0

* Numbers in brackets [#] represent load conditions shown in Figure 3.1-8. A negative sign [-#] indicates that all lateral load effects act in the direction opposite that shown in the figure.

3.1.5.4.4 Member Forces

For this portion of the analysis, the earthquake shears in the girders along Gridline 1 are computed. This analysis considers only 100 percent of the X-direction forces applied in combination with 30 percent of the (positive or negative) Y-direction forces. The X-direction forces are applied with a 5 percent accidental eccentricity to produces a clockwise rotation of the floor plates. The Y-direction forces are applied without eccentricity.

The results of the member force analysis are shown in Figure 3.1-9. In a later part of this example, the girder shears are compared to those obtained from modal-response-spectrum and modal-time-history analyses.

			8.31	9.54	9.07		
R-12			16.1	17.6	17.1		
12-11			25.8	26.3	26.9		
11-10			31.2	31.0	32.9		
10-9			32.7	32.7	30.4	28.9	12.5
9-8			34.5	34.1	32.3	36.0	22.4
8-7			39.1	38.1	36.5	39.2	24.2
7-6			40.4	38.4	37.2	39.6	24.8
6-5	13.1	30.0	31.7	34.3	33.1	34.9	22.2
5-4	22.1	33.6	29.1	31.0	30.1	31.6	20.4
4-3	22.0	33.0	30.5	31.7	31.1	32.2	21.4
3-2	20.9	33.0	30.9	31.8	31.1	32.4	20.4
2-G							

Figure 3.1-9 Seismic shears in girders (kips) as computed using ELF analysis. Analysis includes orthogonal loading and accidental torsion. (1.0 kip = 4.45 kn)

3.1.6 Modal-Response-Spectrum Analysis

The first step in the modal-response-spectrum analysis is the computation of the structural mode shapes and associated periods of vibration. Using the Table 3.1-4 structural masses and the same mathematical model as used for the ELF and the Rayleigh analyses, the mode shapes and frequencies are automatically computed by SAP2000.

The computed periods of vibration for the first 10 modes are summarized in Table 3.1-13, which also shows values called the modal direction factor for each mode. Note that the longest period, 2.867 seconds, is significantly greater than C_uT_a = 2.23 seconds. Therefore, displacements, drift, and member forces as computed from the true modal properties may have to be scaled up to a value consistent with 85 percent of the ELF base shear using $T = C_uT_a$. The smallest period shown in Table 3.1-13 is 0.427 seconds.

The modal direction factors shown in Table 3.1-13 are indices that quantify the direction of the mode. A direction factor of 100.0 in any particular direction would indicate that this mode responds entirely along one of the orthogonal (X, Y or θ_Z axes) of the structure.[3] As Table 3.1-13 shows, the first mode is predominantly X translation, the second mode is primarily Y translation, and the third mode is largely

[3]It should be emphasized that, in general, the principal direction of structural response will not coincide with one of the axes used to describe the structure in three-dimensional space.

torsional. Modes 4 and 5 also are nearly unidirectional, but Modes 6 through 10 have significant lateral-torsional coupling. Plots showing the first eight mode shapes are given in Figure 3.1-10.

It is interesting to note that the X-direction Rayleigh period (2.87 seconds) is virtually identical to the first mode predominately X-direction period (2.867 seconds) computed from the eigenvalue analysis. Similarly, the Y-direction Rayleigh period (2.73 seconds) is very close to second mode predominantly Y-direction period (2.744 seconds) from the eigenvalue analysis. The closeness of the Rayleigh and eigenvalue periods of this building arises from the fact that the first and seconds modes of vibration act primarily along the orthogonal axes. Had the first and second modes not acted along the orthogonal axes, the Rayleigh periods (based on loads and displacements in the X and Y directions) would have been somewhat less accurate.

In Table 3.1-14, the effective mass in Modes 1 through 10 is given as a percentage of total mass. The values shown in parentheses in Table 3.1-14 are the accumulated effective masses and should total 100 percent of the total mass when all modes are considered. By Mode 10, the accumulated effective mass value is approximately 80 percent of the total mass for the translational modes and 72 percent of the total mass for the torsional mode. *Provisions* Sec. 5.5.2 [5.3.2] requires that a sufficient number of modes be represented to capture at least 90 percent of the total mass of the structure. On first glance, it would seem that the use of 10 modes as shown in Table 3.1-14 violates this rule. However, approximately 18 percent of the total mass for this structure is located at grade level and, as this level is extremely stiff, this mass does not show up as an effective mass until Modes 37, 38, and 39 are considered. In the case of the building modeled as a 13-story building with a very stiff first story, the accumulated 80 percent of effective translational mass in Mode 10 actually represents almost 100 percent of the dynamically excitable mass. In this sense, the *Provisions* requirements are clearly met when using only the first 10 modes in the response spectrum or time-history analysis. For good measure, 14 modes were used in the actual analysis.

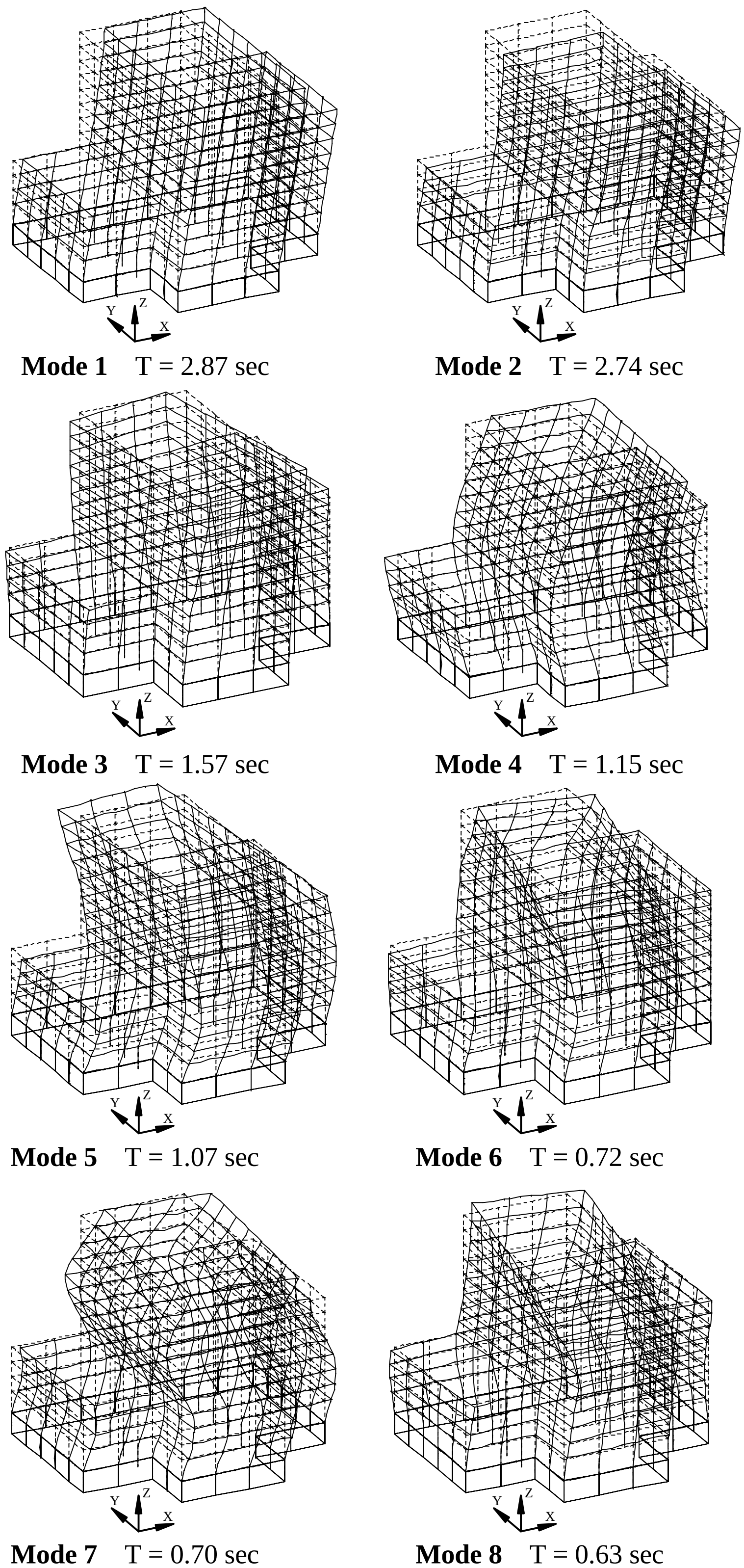


Figure 3.1-10 Mode shapes as computed using SAP2000.

Table 3.1-13 Computed Periods and Direction Factors

Mode	Period (seconds)	Modal Direction Factor		
		X Translation	Y Translation	Z Torsion
1	2.867	99.2	0.7	0.1
2	2.745	0.8	99.0	0.2
3	1.565	1.7	9.6	88.7
4	1.149	98.2	0.8	1.0
5	1.074	0.4	92.1	7.5
6	0.724	7.9	44.4	47.7
7	0.697	91.7	5.23	3.12
8	0.631	0.3	50.0	49.7
9	0.434	30.0	5.7	64.3
10	0.427	70.3	2.0	27.7

Table 3.1-14 Computed Periods and Effective Mass Factors

Mode	Period (seconds)	Effective Mass Factor		
		X Translation	Y Translation	Z Torsion
1	2.867	64.04 (64.0)	0.46 (0.5)	0.04 (0.0)
2	2.744	0.51 (64.6)	64.25 (64.7)	0.02 (0.1)
3	1.565	0.34 (64.9)	0.93 (65.6)	51.06 (51.1)
4	1.149	10.78 (75.7)	0.07 (65.7)	0.46 (51.6)
5	1.074	0.04 (75.7)	10.64 (76.3)	5.30 (56.9)
6	0.724	0.23 (75.9)	1.08 (77.4)	2.96 (59.8)
7	0.697	2.94 (78.9)	0.15 (77.6)	0.03 (59.9)
8	0.631	0.01 (78.9)	1.43 (79.0)	8.93 (68.8)
9	0.434	0.38 (79.3)	0.00 (79.0)	3.32 (71.1)
10	0.427	1.37 (80.6)	0.01 (79.0)	1.15 (72.3)

3.1.6.1 Response Spectrum Coordinates and Computation of Modal Forces

The coordinates of the response spectrum are based on *Provisions* Eq. 4.1.2.6-1 and 4.1.2.6-2 [3.3-5 and 3.3-6]. [In the 2003 *Provisions*, the design response spectrum has reduced ordinates at very long periods as indicated in Sec. 3.3.4. The new portion of the spectrum reflects a constant ground displacement at periods greater than T_L, the value of which is based on the magnitude of the source earthquake that dominates the probabilistic ground motion at the site.]

For periods less than T_0:

$$S_a = 0.6\frac{S_{DS}}{T_0}T + 0.4S_{DS}$$

and for periods greater than T_S:

$$S_a = \frac{S_{D1}}{T}$$

where $T_0 = 0.2 S_{DS} / S_{D1}$ and $T_S = S_{D1} / S_{DS}$.

Using S_{DS} = 0.833 and S_{D1} = 0.373, T_0 = 0.089 seconds and T_S = 0.448 seconds. The computed response-spectrum coordinates for several period values are shown in Table 3.1-15 and the response spectrum, shown with and without the *I/R* =1/8 modification, is plotted in Figure 3.1-11. The spectrum does not include the high period limit on C_s ($C_{s,\,min}$ = 0.044IS_{DS}), which controlled the ELF base shear for this structure and which ultimately will control the scaling of the results from the response-spectrum analysis. (Recall that if the computed base shear falls below 85 percent of the ELF base shear, the computed response must be scaled up such that the computed base shear equals 85 percent of the ELF base shear.)

Table 3.1-15 Response Spectrum Coordinates

T_m (seconds)	C_{sm}	$C_{sm}(I/R)$
0.000	0.333	0.0416
0.089 (T_0)	0.833	0.104
0.448 (T_S)	0.833	0.104
1.000	0.373	0.0446
1.500	0.249	0.0311
2.000	0.186	0.0235
2.500	0.149	0.0186
3.000	0.124	0.0155

I = 1, *R* = 8.7.

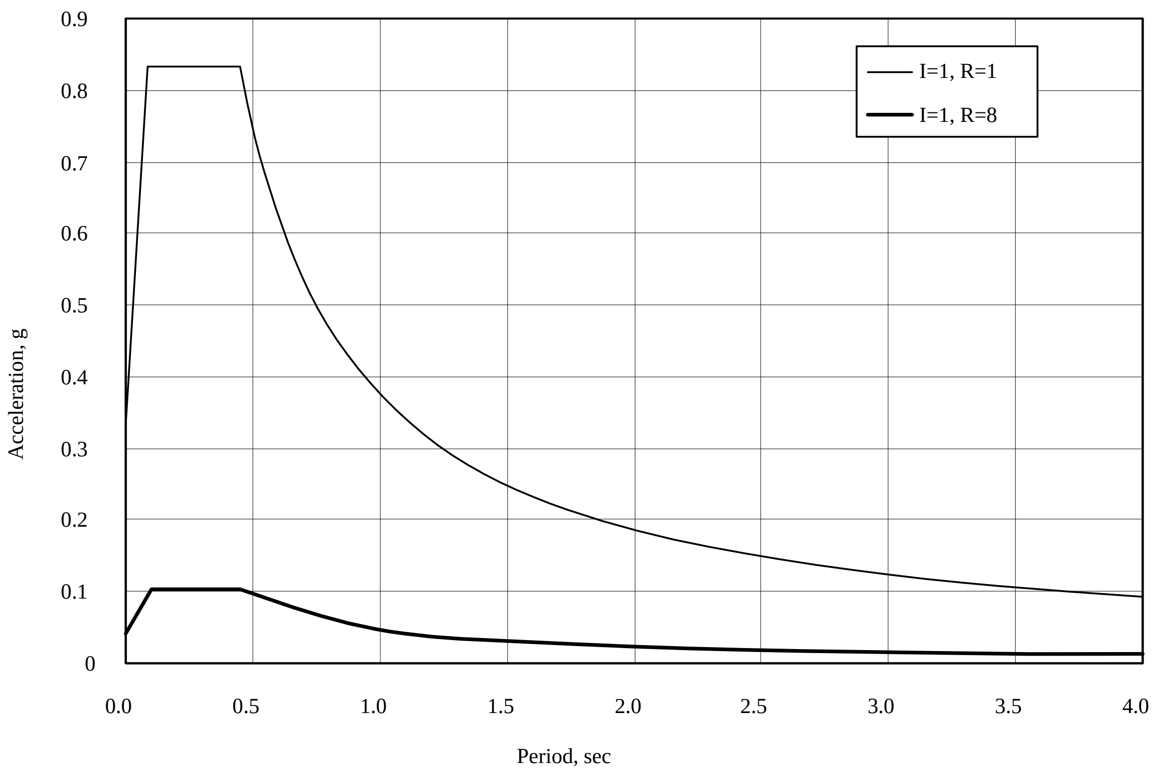


Figure 3.1-11 Total acceleration response spectrum used in analysis.

Using the response spectrum coordinates of Table 3.1-15, the response-spectrum analysis was carried out using SAP2000. As mentioned above, the first 14 modes of response were computed and superimposed using complete quadratic combination (CQC). A modal damping ratio of 5 percent of critical was used in the CQC calculations.

Two analyses were carried out. The first directed the seismic motion along the X axis of the structure, and the second directed the motion along the Y axis. Combinations of these two loadings plus accidental torsion are discussed later. The response spectrum used in the analysis did include *I/R*.

3.1.6.1.1 Dynamic Base Shear

After specifying member "groups," SAP2000 automatically computes and prints the CQC story shears. Groups were defined such that total shears would be printed for each story of the structure. The base shears were printed as follows:

X-direction base shear = 437.7 kips
Y-direction base shear = 454.6 kips

These values are much lower that the ELF base shear of 1124 kips. Recall that the ELF base shear was controlled by *Provisions* Eq. 5.4.1.1-3. The modal-response-spectrum shears are less than the ELF shears because the fundamental period of the structure used in the response-spectrum analysis is 2.87 seconds (vs 2.23) and because the response spectrum of Figure 3.1-11 does not include the minimum base shear limit imposed by *Provisions* Eq. 5.4.1.1-3. [Recall that the equation for minimum base shear coefficient does not appear in the 2003 *Provisions*.]

According to *Provisions* Sec. 5.5.7 [5.3.7], the base shears from the modal-response-spectrum analysis must not be less than 85 percent of that computed from the ELF analysis. If the response spectrum shears are lower than the ELF shear, then the computed shears and displacements must be scaled up such that the response spectrum base shear is 85 percent of that computed from the ELF analysis.

Hence, the required scale factors are:

X-direction scale factor = 0.85(1124)/437.7 = 2.18
Y-direction scale factor = 0.85(1124)/454.6 = 2.10

The computed and scaled story shears are as shown in Table 3.1-16. Since the base shears for the ELF and the modal analysis are different (due to the 0.85 factor), direct comparisons cannot be made between Table 3.1-11 and Table 3.1-4. However, it is clear that the vertical distribution of forces is somewhat similar when computed by ELF and modal-response spectrum.

Table 3.1-16 Story Shears from Modal-Response-Spectrum Analysis

Story	X Direction (SF = 2.18)		Y Direction (SF = 2.10)	
	Unscaled Shear (kips)	Scaled Shear (kips)	Unscaled Shear (kips)	Scaled Shear (kips)
R-12	82.5	180	79.2	167
12-11	131.0	286	127.6	268
11-10	163.7	358	163.5	344
10-9	191.1	417	195.0	410
9-8	239.6	523	247.6	521
8-7	268.4	586	277.2	583
7-6	292.5	638	302.1	635
6-5	315.2	688	326.0	686
5-4	358.6	783	371.8	782
4-3	383.9	838	400.5	843
3-2	409.4	894	426.2	897
2-G	437.7	956	454.6	956

1.0 kip = 4.45 kN.

3.1.6.2 Drift and P-Delta Effects

According to *Provisions* Sec. 5.5.7 [5.3.7], the computed displacements and drift (as based on the response spectrum of Figure 3.1-11) must also be scaled by the base shear factors (SF) of 2.18 and 2.10 for the structure loaded in the X and Y directions, respectively.

In Tables 3.1-17 and 3.1-18, the story displacement from the response-spectrum analysis, the scaled story displacement, the scaled story drift, the amplified story drift (as multiplied by C_d = 5.5), and the allowable story drift are listed. As may be observed from the tables, the allowable drift is not exceeded at any level.

P-delta effects are computed for the X-direction response as shown in Table 3.1-19. Note that the scaled story shears from Table 3.1-16 are used in association with the scaled story drifts (including C_d) from Table 3.1-17. The story stability factors are above the limit (θ_{max} = 0.091) only at the bottom two levels of the structure and are only marginally above the limit. As the β factor was conservatively set at 1.0 in computing the limit, it is likely that a refined analysis for β would indicate that P-delta effects are not of particular concern for this structure.

Table 3.1-17 Response Spectrum Drift for Building Responding in X Direction

Level	1 Total Drift from R.S. Analysis (in.)	2 Scaled Total Drift [Col-1 × 2.18] (in.)	3 Scaled Drift (in.)	4 Scaled Story Drift × C_d (in.)	5 Allowable Story Drift (in.)
R	1.96	4.28	0.18	0.99	3.00
12	1.88	4.10	0.26	1.43	3.00
11	1.76	3.84	0.30	1.65	3.00
10	1.62	3.54	0.33	1.82	3.00
9	1.47	3.21	0.34	1.87	3.00
8	1.32	2.87	0.36	1.98	3.00
7	1.15	2.51	0.40	2.20	3.00
6	0.968	2.11	0.39	2.14	3.00
5	0.789	1.72	0.38	2.09	3.00
4	0.615	1.34	0.38	2.09	3.00
3	0.439	0.958	0.42	2.31	3.00
2	0.245	0.534	0.53	2.91	4.32

1.0 in. = 25.4 mm.

Table 3.1-18 Spectrum Response Drift for Building Responding in Y Direction

Level	1 Total Drift from R.S. Analysis (in.)	2 Scaled Total Drift [Col-1 × 2.18] (in.)	3 Scaled Drift (in.)	4 Scaled Story Drift × C_d (in.)	5 Allowable Story Drift (in.)
R	1.84	3.87	0.12	0.66	3.00
12	1.79	3.75	0.20	1.10	3.00
11	1.69	3.55	0.24	1.32	3.00
10	1.58	3.31	0.37	2.04	3.00
9	1.40	2.94	0.29	1.60	3.00
8	1.26	2.65	0.33	1.82	3.00
7	1.10	2.32	0.35	1.93	3.00
6	0.938	1.97	0.38	2.09	3.00
5	0.757	1.59	0.32	1.76	3.00
4	0.605	1.27	0.36	2.00	3.00
3	0.432	0.908	0.39	2.14	3.00
2	0.247	0.518	0.52	2.86	4.32

1.0 in. = 25.4 mm.

Table 3.1-19 Computation of P-Delta Effects for X-Direction Response

Level	h_{sx} (in.)	Δ (in.)	P_D (kips)	P_L (kips)	P_T (kips)	P_X (kips)	V_X (kips)	θ_X
R	150	0.99	1656.5	315.0	1971.5	1971.5	180	0.013
12	150	1.43	1595.8	315.0	1910.8	3882.3	286	0.024
11	150	1.65	1595.8	315.0	1910.8	5793.1	358	0.032
10	150	1.82	1595.8	315.0	1910.8	7703.9	417	0.041
9	150	1.87	3403.0	465.0	3868.0	11571.9	523	0.050
8	150	1.98	2330.8	465.0	2795.8	14367.7	586	0.059
7	150	2.20	2330.8	465.0	2795.8	17163.5	638	0.072
6	150	2.14	2330.8	465.0	2795.8	19959.3	688	0.075
5	150	2.09	4323.8	615.0	4938.8	24898.1	783	0.081
4	150	2.09	3066.1	615.0	3681.1	28579.2	838	0.086
3	150	2.31	3066.1	615.0	3681.1	32260.3	894	0.101
2	216	2.91	3097.0	615.0	3712.0	35972.3	956	0.092

1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.

3.1.6.3 Torsion, Orthogonal Loading, and Load Combinations

To determine member design forces, it is necessary to add the effects of accidental torsion and orthogonal loading into the analysis. When including accidental torsion in modal-response-spectrum analysis, there are generally two approaches that can be taken:

1. Displace the center of mass of the floor plate plus or minus 5 percent of the plate dimension perpendicular to the direction of the applied response spectrum. As there are four possible mass locations, this will require four separate modal analyses for torsion with each analysis using a different set of mode shapes and frequencies.

2. Compute the effects of accidental torsion by creating a load condition with the story torques applied as static forces. Member forces created by the accidental torsion are then added directly to the results of the response-spectrum analysis. Since the sign of member forces in the response-spectrum analysis is lost as a result of SRSS or CQC combinations, the absolute value of the member forces resulting from accidental torsion should be used. As with the displaced mass method, there are four possible ways to apply the accidental torsion: plus and minus torsion for primary loads in the X or Y directions. Because of the required scaling, the static torsional forces should be based on 85 percent of the ELF forces.

Each of the above approaches has advantages and disadvantages. The primary disadvantage of the first approach is a practical one: most computer programs do not allow for the extraction of member force maxima from more than one run when the different runs incorporate a different set of mode shapes and frequencies. For structures that are torsionally regular and will not require amplification of torsion, the second approach is preferred. For torsionally flexible structures, the first approach may be preferred because the dynamic analysis will automatically amplify the torsional effects. In the analysis that follows, the second approach has been used because the structure has essentially rigid diaphragms and high torsional rigidity and amplification of accidental torsion is not required.

There are three possible methods for applying the orthogonal loading rule:

1. Run the response-spectrum analysis with 100 percent of the scaled X spectrum acting in one direction, concurrent with the application of 30 percent of the scaled Y spectrum acting in the orthogonal direction. Use CQC for combining modal maxima. Perform a similar analysis for the larger seismic forces acting in the Y direction.

2. Run two separate response-spectrum analyses, one in the X direction and one in the Y direction, with CQC being used for modal combinations in each analysis. Using a direct sum, combine 100 percent of the scaled X-direction results with 30 percent of the scaled Y-direction results. Perform a similar analysis for the larger loads acting in the Y direction.

3. Run two separate response-spectrum analyses, one in the X direction and one in the Y-direction, with CQC being used for modal combinations in each analysis. Using SRSS, combine 100 percent of the scaled X-direction results with 100 percent of the scaled Y-direction results.[4]

All seismic effects can be considered in only two load cases by using Approach 2 for accidental torsion and Approach 2 for orthogonal loading. These are shown in Figure 3.1-12. When the load combinations required by *Provisions* Sec. 5.2.7 [4.2.2.1] are included, the total number of load combinations will double to four.

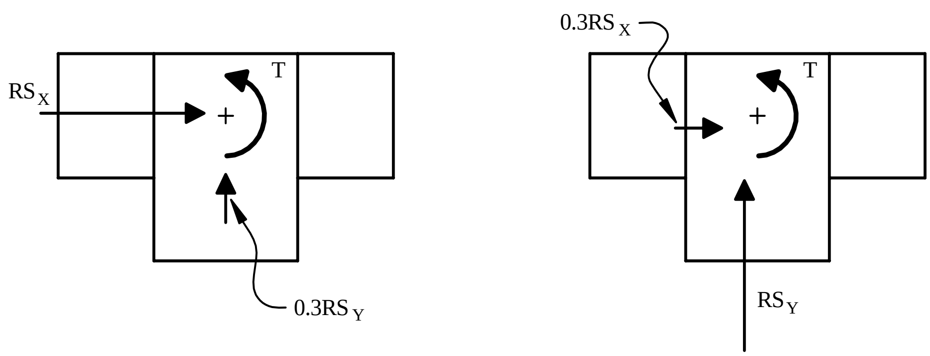


Figure 3.1-12 Load combinations from response-spectrum analysis.

3.1.6.4 Member Design Forces

Earthquake shear forces in the beams of Frame 1 are given in Figure 3.1-13 for the X direction of response. These forces include 100 percent of the scaled X-direction spectrum added to the 30 percent of the scaled Y-direction spectrum. Accidental torsion is then added to the combined spectral loading. The design force for the Level 12 beam in Bay 3 (shown in bold type in Figure 3.1-13) was computed as follows:

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[4]This method has been forwarded in the unpublished paper *A Seismic Analysis Method Which Satisfies the 1988 UBC Lateral Force Requirements*, written in 1989 by Wilson, Suharwardy, and Habibullah. The paper also suggests the use of a single scale factor, where the scale factor is based on the total base shear developed along the principal axes of the structure. As stated in the paper, the major advantage of the method is that one set of dynamic design forces, including the effect of accidental torsion, is produced in one computer run. In addition, the resulting structural design has equal resistance to seismic motions in all possible directions.

Force from 100 percent X-direction spectrum = 6.94 kips (as based on CQC combination for structure loaded with X spectrum only).

Force from 100 percent Y-direction spectrum = 1.26 kips (as based on CQC combination for structure loaded with Y spectrum only).

Force from accidental torsion = 1.25 kips.
Scale factor for X-direction response = 2.18.
Scale factor for Y-direction response = 2.10.

Earthquake shear force = (2.18 × 6.94) + (2.10 × 0.30 × 1.26) + (0.85 × 1.25) = 17.0 kips

			9.4	9.7	9.9		
R-12			**17.0**	17.7	17.8		
12-11			25.0	24.9	26.0		
11-10			28.2	27.7	29.8		
10-9			26.6	26.5	24.8	22.9	10.2
9-8			27.2	26.7	25.5	28.0	18.0
8-7			30.9	28.8	28.8	30.5	19.4
7-6			32.3	30.4	29.8	31.1	20.1
6-5	11.1	24.4	26.0	27.7	27.1	27.9	18.6
5-4	19.0	28.8	25.7	27.0	26.6	27.1	18.6
4-3	20.1	29.7	28.0	28.8	28.4	29.0	20.2
3-2	20.0	31.5	30.1	30.6	30.4	31.1	20.1
2-G							

Figure 3.1-13 Seismic shears in girders (kips) as computed using response-spectrum analysis. Analysis includes orthogonal loading and accidental torsion (1.0 kip = 4.45 kN).

3.1.7 Modal-Time-History Analysis

In modal-time-history analysis, the response in each mode is computed using step-by-step integration of the equations of motion, the modal responses are transformed to the structural coordinate system, linearly superimposed, and then used to compute structural displacements and member forces. The displacement and member forces for each time step in the analysis or minimum and maximum quantities (response envelopes) may be printed.

Requirements for time-history analysis are provided in *Provisions* Sec. 5.6 [5.4]. The same mathematical model of the structure used for the ELF and response-spectrum analysis is used for the time-history analysis.

As allowed by *Provisions* Sec. 5.6.2 [5.4.2], the structure will be analyzed using three different pairs of ground motion time-histories. The development of a proper suite of ground motions is one of the most critical and difficult aspects of time-history approaches. The motions should be characteristic of the site and should be from real (or simulated) ground motions that have a magnitude, distance, and source mechanism consistent with those that control the maximum considered earthquake.

For the purposes of this example, however, the emphasis is on the *implementation* of the time-history approach rather than on selection of realistic ground motions. For this reason, the motion suite developed for Example 3.2 is also used for the present example.[5] The structure for Example 3.2 is situated in Seattle, Washington, and uses three pairs of motions developed specifically for the site. The use of the Seattle motions for a Stockton building analysis is, of course, not strictly consistent with the requirements of the *Provisions*. However, a realistic comparison may still be made between the ELF, response spectrum, and time-history approaches.

3.1.7.1 The Seattle Ground Motion Suite

It is beneficial to provide some basic information on the Seattle motion suites in Table 3.1-20 below. Refer to Figures 3.2-40 through 3.2-42 for additional information, including plots of the ground motion time histories and 5-percent-damped response spectra for each motion.

Table 3.1-20 Seattle Ground Motion Parameters (Unscaled)

Record Name	Orientation	Number of Points and Time Increment	Peak Ground Acceleration (g)	Source Motion
Record A00	N-S	8192 @ 0.005 seconds	0.443	Lucern (Landers)
Record A90	E-W	8192 @ 0.005 seconds	0.454	Lucern (Landers)
Record B00	N-S	4096 @ 0.005 seconds	0.460	USC Lick (Loma Prieta)
Record B90	E-W	4096 @ 0.005 seconds	0.435	USC Lick (Loma Prieta)
Record C00	N-S	1024 @ 0.02 seconds	0.460	Dayhook (Tabas, Iran)
Record C90	E-W	1024 @ 0.02 seconds	0.407	Dayhook (Tabas, Iran)

Before the ground motions may be used in the time-history analysis, they must be scaled using the procedure described in *Provisions* Sec. 5.6.2.2 [5.4.2.2]. One scale factor will be determined for each pair of ground motions. The scale factors for record sets A, B, and C will be called S_A, S_B, and S_C, respectively.

The scaling process proceeds as follows:

1. For each pair of motions (A, B, and C):
   a Assume an initial scale factor (S_A, S_B, S_C),
   b. Compute the 5-percent-damped elastic response spectrum for each component in the pair,
   c. Compute the SRSS of the spectra for the two components, and
   d. Scale the SRSS using the factor from (a) above.

2. Adjust scale factors (S_A, S_B, and S_C) such that the average of the three scaled SRSS spectra over the period range $0.2T_1$ to 1.5 T_1 is not less than 1.3 times the 5-percent-damped spectrum determined in accordance with *Provisions* Sec. 4.1.2.6 [3.3.4]. T_1 is the fundamental mode period of vibration of

[5] See Sec. 3.2.6.2 of this volume of design examples for a detailed discussion of the selected ground motions.

the structure. (The factor of 1.3 more than compensates for the fact that taking the SRSS of the two components of a ground motion effectively increases their magnitude by a factor of 1.414.)

Note that the scale factors so determined are not unique. An infinite number of different scale factors will satisfy the above requirements, and it is up to the engineer to make sure that the selected scale factors are reasonable.[6] Because the original ground motions are similar in terms of peak ground acceleration, the same scale factor will be used for each motion; hence, $S_A = S_B = S_C$. This equality in scale factors would not necessarily be appropriate for other suites of motions.

Given the 5-percent-damped spectra of the ground motions, this process is best carried out using an Excel spreadsheet. The spectra themselves were computed using the program *NONLIN*.[7] The results of the analysis are shown in Figures 3.1-14 and 3.1-15. Figure 3.1-14 shows the average of the SRSS of the *unscaled* spectra together with the *Provisions* response spectrum using S_{DS} = 0.833g (322 in./sec^2) and S_{D1} = 0.373g (144 in./sec^2). Figure 3.1-15 shows the ratio of the average SRSS spectrum to the *Provisions* spectrum over the period range 0.573 seconds to 4.30 seconds, where a scale factor $S_A = S_B = S_C = 0.922$ has been applied to each original spectrum. As can be seen, the minimum ratio of 1.3 occurs at a period of approximately 3.8 seconds.

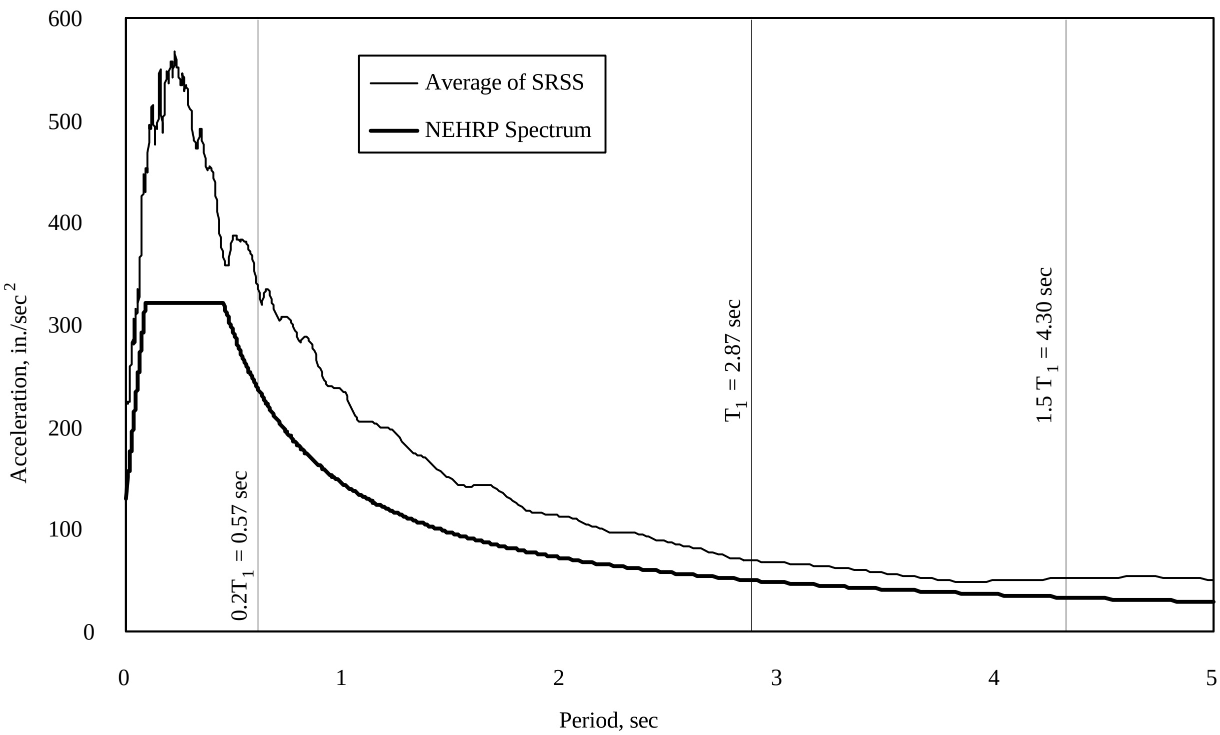

Figure 3.1-14 Unscaled SRSS of spectra of ground motion pairs together with *Provisions* spectrum (1.0 in. = 25.4 mm).

At all other periods, the effect of using the 0.922 scale factor to provide a minimum ratio of 1.3 over the target period range is to have a relatively higher scale factor at all other periods if those periods significantly contribute to the response. For example, at the structure's fundamental mode, with T = 2.867 sec, the ratio of the scaled average SRSS to the *Provisions* spectrum is 1.38, not 1.30. At the higher modes, the effect is even more pronounced. For example, at the second translational X mode, T = 1.149

[6] The "degree of freedom" in selecting the scaling factors may be used to reduce the effect of a particularly demanding motion.

[7] *NONLIN*, developed by Finley Charney, may be downloaded at no cost at www.fema.gov/emi. To find the latest version, do a search for *NONLIN*.

seconds and the computed ratio is 1.62. This, of course, is an inherent difficulty of using a single scale factor to scale ground motion spectra to a target code spectrum.

When performing linear-time-history analysis, the ground motions also should be scaled by the factor *I/R*. In this case, I = 1 and R = 8, so the actual scale factor applied to each ground motion will be 0.922(1/8) = 0.115.

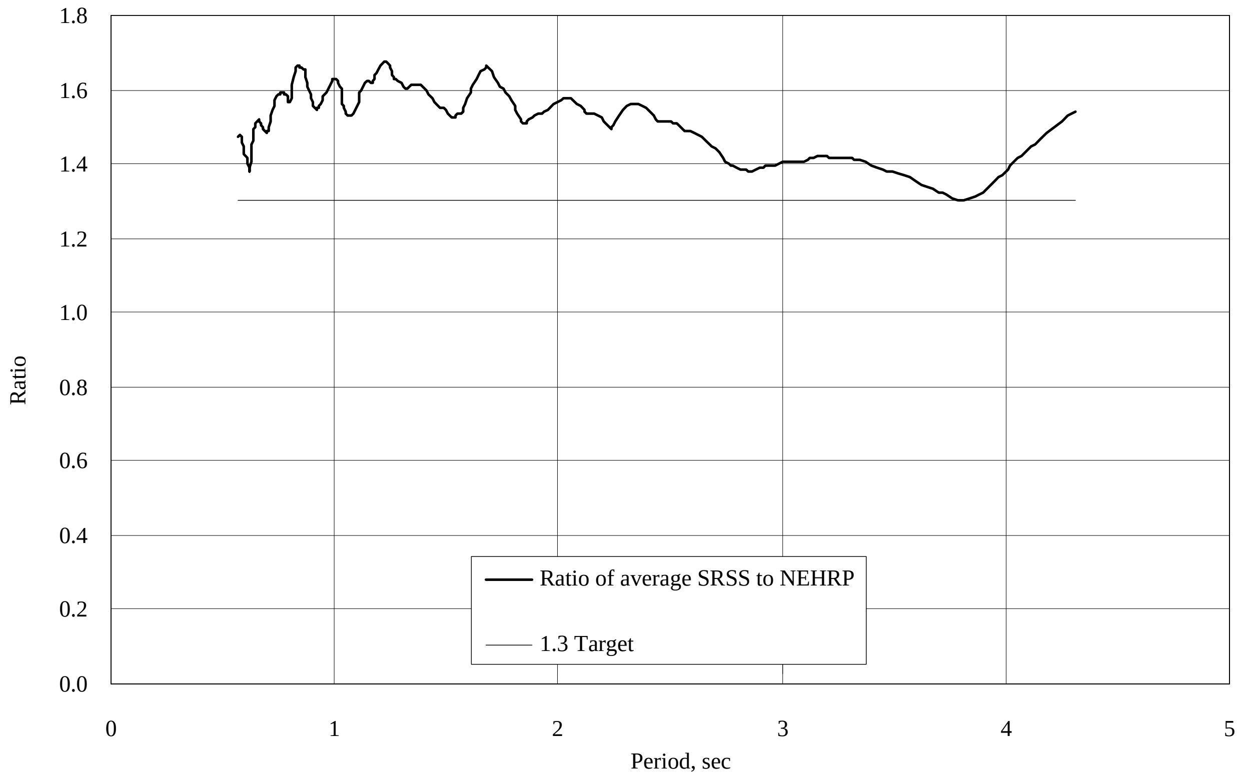

Figure 3.1-15 Ratio of average scaled SRSS spectrum to *Provisions* spectrum.

If the maximum base shear from any of the analyses is less than that computed from *Provisions* Eq. 5.4.1.1-3 ($C_s = 0.044IS_{DS}$), all forces and displacements[8] computed from the time-history analysis must again be scaled such that peak base shear from the time-history analysis is equal to the minimum shear computed from Eq. 5.4.1.1-3. This is stated in *Provisions* Sec. 5.6.3 [5.4.3]. Recall that the base shear controlled by Eq. 5.4.1.1-3 is 1124 kips in each direction. [In the 2003 *Provisions* base shear scaling is still required, but recall that the minimum base shear has been revised.]

The second paragraph of *Provisions* Sec. 5.6.3 [5.4.3] states that if fewer than seven ground motion pairs are used in the analysis, the design of the structure should be based on the maximum scaled quantity among all analyses.

The *Provisions* is not particularly clear regarding the scaling of displacements in time-history analysis. The first paragraph of Sec. 5.6.3 states that member forces should be scaled, but displacements are not mentioned. The second paragraph states that member forces and displacements should be scaled. In this example, the displacements will be scaled, mainly to be consistent with the response spectrum procedure which, in *Provisions* Sec. 5.5.7, explicitly states that forces and displacements should be scaled. See Sec. 3.1.8 of this volume of design examples for more discussion of this apparent inconsistency in the *Provisions*.

Twelve individual time-history analyses were carried out using SAP2000: one for each N-S ground motion acting in the X direction, one for each N-S motion acting in the Y direction, one for each E-W motion acting in the X direction, and one for each E-W motion acting in the Y direction. As with the response-spectrum analysis, 14 modes were used in the analysis. Five percent of critical damping was used in each mode. The integration time-step used in all analyses was 0.005 seconds. The results from the analyses are summarized Tables 3.1-21 and 3.1-22.

As may be observed from Table 3.1-21, the maximum scaled base shears computed from the time-history analysis are significantly less than the ELF minimum of 1124 kips. This is expected because the ELF base shear was controlled by *Provisions* Eq. 5.4.1.1-3. Hence, each of the analyses will need to be scaled up. The required scale factors are shown in Table 3.1-22. Also shown in that table are the scaled maximum deflections with and without $C_d = 5.5$.

Table 3.1-21 Result Maxima from Time-History Analysis (Unscaled)

Analysis	Maximum Base Shear (S.F. = 0.115) (kips)	Time of Maximum Shear (sec)	Maximum Roof Displacement (S.F. = 0.115) (in.)	Time of Maximum Displacement (sec)
A00-X	394.5	12.73	2.28	11.39
A00-Y	398.2	11.84	2.11	11.36
A90-X	473.8	15.42	2.13	12.77
A90-Y	523.9	15.12	1.91	10.90
B00-X	393.5	15.35	2.11	14.17
B00-Y	475.1	14.29	1.91	19.43
B90-X	399.6	13.31	1.77	16.27
B90-Y	454.2	12.83	1.68	12.80
C00-X	403.1	6.96	1.86	7.02
C00-Y	519.2	6.96	1.70	7.02
C90-X	381.5	19.40	1.95	19.38
C90-Y	388.5	19.38	1.85	19.30

1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.

Table 3.1-22 Result Maxima from Time-History Analysis (Scaled)

Analysis	Maximum Base Shear (SF = 0.115) (kips)	Required Additional Scale Factor for V = 1124 kips	Adjusted Maximum Roof Displacement (in.)	Adjusted Max Roof Disp. × C_d (in.)
A00-X	394.5	2.85	6.51	35.7
A00-Y	398.2	2.82	5.95	32.7
A90-X	473.8	2.37	5.05	27.8
A90-Y	523.9	2.15	4.11	22.6
B00-X	393.5	2.86	6.03	33.2
B00-Y	475.1	2.37	4.53	24.9
B90-X	399.6	2.81	4.97	27.4
B90-Y	454.2	2.48	4.17	22.9
C00-X	403.1	2.79	5.19	28.5
C00-Y	519.2	2.16	3.67	20.2
C90-X	381.5	2.95	5.75	31.6
C90-Y	388.5	2.89	5.35	29.4

Scaled base shear = 1124 kips for all cases. 1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.

3.1.7.2 Drift and P-Delta Effects

In this section, drift and P-delta effects are checked only for the structure subjected to Motion A00 acting in the X direction of the building. As can be seen from Table 3.1-22, this analysis produced the largest roof displacement, but not necessarily the maximum story drift. To be sure that the maximum drift has been determined, it would be necessary to compute the scaled drifts histories from each analysis and then find the maximum drift among all analyses.

As may be observed from Table 3.1-23, the allowable drift has been exceeded at several levels. For example, at Level 11, the computed drift is 4.14 in. compared to the limit of 3.00 inches.

Before computing P-delta effects, it is necessary to determine the story shears that exist at the time of maximum displacement. These shears, together with the inertial story forces, are shown in the first two columns of Table 3.1-24. The maximum base shear at the time of maximum displacement is only 668.9 kips, significantly less that the peak base shear of 1124 kips. For comparison purposes, Table 3.1-24 also shows the story shears and inertial forces that occur at the time of peak base shear.

As may be seen from Table 3.1-25, the P-delta effects are marginally exceeded at the lower three levels of the structure, as the maximum allowable stability ratio for the structure is 0.091 (see Sec. 3.1.5.3 of this example). As mentioned earlier, the fact that the limit has been exceeded is probably of no concern because the factor β was conservatively taken as 1.0.

Table 3.1-23 Time-History Drift for Building Responding in X Direction to Motion A00X

Level	1 Elastic Total Drift (in.)	2 Elastic Story Drift (in.)	3 Inelastic Story Drift (in.)	4 Allowable Drift (in.)
R	6.51	0.47	2.57	3.00
12	6.05	0.66	3.63	3.00
11	5.39	0.75	4.14	3.00
10	4.63	0.75	4.12	3.00
9	3.88	0.62	3.40	3.00
8	3.27	0.61	3.34	3.00
7	2.66	0.58	3.20	3.00
6	2.08	0.54	2.95	3.00
5	1.54	0.42	2.32	3.00
4	1.12	0.39	2.12	3.00
3	0.74	0.34	1.89	3.00
2	0.39	0.39	2.13	4.32

Computations are at time of maximum roof displacement from analysis A00X. 1.0 in. = 25.4 mm.

Table 3.1-24 Scaled Inertial Force and Story Shear Envelopes from Analysis A00X

Level	At Time of Maximum Roof Displacement (T = 11.39 sec)		At Time of Maximum Base Shear (T = 12.73 sec)	
	Story Shear (kips)	Inertial Force (kips)	Story Shear (kips)	Inertial Force (kips)
R	307.4	307.4	40.2	40.2
12	529.7	222.3	44.3	4.1
11	664.9	135.2	45.7	1.4
10	730.5	65.6	95.6	49.9
9	787.9	57.4	319.0	223.4
8	817.5	29.6	468.1	149.1
7	843.8	26.3	559.2	91.1
6	855.0	11.2	596.5	37.3
5	828.7	-26.3	662.7	66.2
4	778.7	-50.0	785.5	122.8
3	716.1	-62.6	971.7	186.2
2	668.9	-47.2	1124.0	148.3

1.0 kip = 4.45 kN.

Table 3.1-25 Computation of P-Delta Effects for X-Direction Response

Level	h_{sx} (in.)	Δ (in.)	P_D (kips)	P_L (kips)	P_T (kips)	P_X (kips)	V_X(kips)	θ_X
R	150	2.57	1656.5	315.0	1971.5	1971.5	307.4	0.020
12	150	3.63	1595.8	315.0	1910.8	3882.3	529.7	0.032
11	150	4.14	1595.8	315.0	1910.8	5793.1	664.9	0.044
10	150	4.12	1595.8	315.0	1910.8	7703.9	730.5	0.053
9	150	3.40	3403.0	465.0	3868.0	11571.9	787.9	0.061
8	150	3.34	2330.8	465.0	2795.8	14367.7	817.5	0.071
7	150	3.20	2330.8	465.0	2795.8	17163.5	843.8	0.079
6	150	2.95	2330.8	465.0	2795.8	19959.3	855.0	0.083
5	150	2.32	4323.8	615.0	4938.8	24898.1	828.7	0.084
4	150	2.12	3066.1	615.0	3681.1	28579.2	778.7	0.094
3	150	1.89	3066.1	615.0	3681.1	32260.3	716.1	0.103
2	216	2.13	3097.0	615.0	3712.0	35972.3	668.9	0.096

Computations are at time of maximum roof displacement from analysis A00X. 1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.

3.1.7.3 Torsion, Orthogonal Loading, and Load Combinations

As with ELF or response-spectrum analysis, it is necessary to add the effects of accidental torsion and orthogonal loading into the analysis. Accidental torsion is applied in exactly the same manner as done for the response spectrum approach, except that the factor 0.85 is not used. Orthogonal loading is automatically accounted for by concurrently running one ground motion in one principal direction with

30 percent of the companion motion being applied in the orthogonal direction. Because the signs of the ground motions are arbitrary, it is appropriate to add the absolute values of the responses from the two directions. Six dynamic load combinations result:

Combination 1:	A00X + 0.3 A90Y + Torsion
Combination 2:	A90X + 0.3 A00Y + Torsion
Combination 3:	B00X + 0.3 B90Y + Torsion
Combination 4:	B90X + 0.3 B00Y + Torsion
Combination 5:	C00X + 0.3 C90Y + Torsion
Combination 6:	C90X + 0.3 C00Y + Torsion

3.1.7.4 Member Design Forces

Using the method outlined above, the individual beam shear maxima developed in Fame 1 were computed for each load combination. The envelope values from only the first two combinations are shown in Figure 3.1-16. Envelope values from all combinations are shown in Figure 3.1-17. Note that some of the other combinations (Combinations 3 through 8) control the member shears at the lower levels of the building. These forces are compared to the forces obtained using ELF and modal-response-spectrum analysis in the following discussion.

			16.2	17.5	17.6		
R-12			30.4	32.3	32.3		
12-11			45.5	45.6	47.6		
11-10			50.0	49.3	52.8		
10-9			43.9	44.4	40.9	37.8	16.3
9-8			42.0	41.9	39.6	44.4	28.4
8-7			44.9	44.3	42.1	45.4	28.9
7-6			43.4	42.0	40.2	43.7	27.9
6-5	13.7	30.3	32.3	34.2	33.5	34.9	23.0
5-4	23.2	35.6	31.1	32.9	32.4	33.2	22.8
4-3	23.7	35.6	32.6	34.0	33.6	34.4	24.0
3-2	23.1	36.2	35.1	35.3	35.4	35.8	23.4
2-G							

Figure 3.1-16 For Combinations 1 and 2, beam shears (kips) as computed using time-history analysis; analysis includes orthogonal loading and accidental torsion (1.0 kip = 4.45 kN).

			16.2	17.5	17.6		
R-12			30.4	32.3	32.3		
12-11			45.5	45.6	47.6		
11-10			50.0	49.3	52.8		
10-9			44.7	44.5	41.7	38.5	17.3
9-8			43.9	43.5	41.3	45.8	29.6
8-7			46.6	45.4	43.6	46.7	29.6
7-6			45.2	42.9	41.8	44.1	28.5
6-5	14.9	32.4	34.4	36.4	35.6	36.7	24.2
5-4	24.9	37.9	33.5	35.3	34.8	35.6	24.2
4-3	25.3	37.1	35.6	36.1	36.0	36.2	25.3
3-2	24.6	38.2	36.9	37.3	37.3	37.8	24.6
2-G							

Figure 3.1-17 For all combinations, beam shears (kips) as computed using time-history analysis; analysis includes orthogonal loading and accidental torsion (1.0 kip = 4.45 kN).

3.1.8 Comparison of Results from Various Methods of Analysis

A summary of the results from all of the analyses is provided in Tables 3.1-26 through 3.1-28.

3.1.8.1 Comparison of Base Shear and Story Shear

The maximum story shears are shown In Table 3.1-26. For the time-history analysis, the shears computed at the time of maximum displacement and time of maximum base shear (from analysis A00X only) are provided. Also shown from the time-history analysis is the envelope of story shears computed among all analyses. As may be observed, the shears from ELF and response-spectrum analysis seem to differ primarily on the basis of the factor 0.85 used in scaling the response spectrum results. ELF does, however, produce relatively larger forces at Levels 6 through 10.

The difference between ELF shears and time-history envelope shears is much more pronounced, particularly at the upper levels where time-history analysis gives larger forces. One reason for the difference is that the scaling of the ground motions has greatly increased the contribution of the higher modes of response.

The time-history analysis also gives shears larger than those computed using the response spectrum procedure, particularly for the upper levels.

3.1.8.2 Comparison of Drift

Table 3.1-27 summarizes the drifts computed from each of the analyses. The time-history drifts are from a single analysis, A00X; envelope values would be somewhat greater. As with shear, the ELF and modal-response-spectrum approaches appear to produce similar results, but the drifts from time-history analysis are significantly greater. Aside from the fact that the 0.85 factor is not applied to time-history response, it is not clear why the time-history drifts are as high as they are. One possible explanation is that the drifts are dominated by one particular pulse in one particular ground motion. As mentioned above, it is also possible that the effect of scaling has been to artificially enhance the higher mode response.

3.1.8.3 Comparison Member Forces

The shears developed in Bay D-E of Frame 1 are compared in Table 3.1-28. The shears from the time-history (TH) analysis are envelope values among all analyses, including torsion and orthogonal load effects. The time-history approach produced larger beam shears than the ELF and response spectrum (RS) approaches, particularly in the upper levels of the building. The effect of higher modes on the response is again the likely explanation for the noted differences.

Table 3.1-26 Summary of Results from Various Methods of Analysis: Story Shear

Level	Story Shear (kips)				
	ELF	RS	TH at Time of Maximum Displacement	TH at Time of Maximum Base Shear	TH. at Envelope
R	187	180	307	40.2	325
12	341	286	530	44.3	551
11	471	358	664	45.7	683
10	578	417	731	95.6	743
9	765	523	788	319	930
8	866	586	818	468	975
7	943	638	844	559	964
6	999	688	856	596	957
5	1070	783	829	663	1083
4	1102	838	779	786	1091
3	1118	894	718	972	1045
2	1124	956	669	1124	1124

1.0 kip = 4.45 kN.

Table 3.1-27 Summary of Results from Various Methods of Analysis: Story Drift

Level	X-Direction Drift (in.)		
	ELF	RS	TH
R	0.982	0.99	2.57
12	1.41	1.43	3.63
11	1.75	1.65	4.14
10	1.92	1.82	4.12
9	1.83	1.87	3.40
8	2.00	1.98	3.34
7	2.03	2.20	3.20
6	1.95	2.14	2.95
5	1.70	2.09	2.32
4	1.70	2.09	2.12
3	1.67	2.31	1.89
2	2.02	2.91	2.13

1.0 in. = 25.4 mm.

Table 3.1-28 Summary of Results from Various Methods of Analysis: Beam Shear

Level	Beam Shear Force in Bay D-E of Frame 1 (kips)		
	ELF	RS	TH
R	9.54	9.70	17.5
12	17.6	17.7	32.3
11	26.3	24.9	45.6
10	31.0	27.7	49.3
9	32.7	26.5	44.5
8	34.1	26.7	43.5
7	38.1	28.8	45.4
6	38.4	30.4	42.9
5	34.3	27.7	36.4
4	31.0	27.0	35.3
3	31.7	28.8	36.1
2	31.8	30.6	37.3

1.0 kip = 4.45 kN.

3.1.8.4 A Commentary on the *Provisions* Requirements for Analysis

From the writer's perspective, there are two principal inconsistencies between the requirements for ELF, modal-response-spectrum, and modal-time-history analyses:

1. In ELF analysis, the *Provisions* allows displacements to be computed using base shears consistent with Eq. 5.4.1.4-2 [5.2-3] ($C_s = S_{D1}/T(R/I)$ when Eq. 5.4.1.4-3 ($C_S = 0.044IS_{DS}$) controls for strength. For both modal-response-spectrum analysis and modal time-history analysis, however, the computed

shears and displacements must be scaled if the computed base shear falls below the ELF shear computed using Eq. 5.1.1.1-3. [Because the minimum base shear has been revised in the 2003 *Provisions*, this inconsistency would not affect this example.]

2. The factor of 0.85 is allowed when scaling modal-response-spectrum analysis, but not when scaling time-history results. This penalty for time-history analysis is in addition to the penalty imposed by selecting a scale factor that is controlled by the response at one particular period (and thus exceeding the target at other periods). [In the 2003 *Provisions* these inconsistencies are partially resolved. The minimum base shear has been revised, but time-history analysis results are still scaled to a higher base shear than are modal response spectrum analysis results.]

The effect of these inconsistencies is evident in the results shown in Tables 3.1-26 through 3.1-28 and should be addressed prior to finalizing the 2003 edition of the *Provisions*.

3.1.8.5 Which Method Is Best?

In this example, an analysis of an irregular steel moment frame was performed using three different techniques: equivalent-lateral-force, modal-response-spectrum, and modal-time-history analyses. Each analysis was performed using a linear elastic model of the structure even though it is recognized that the structure will repeatedly yield during the earthquake. Hence, each analysis has significant shortcomings with respect to providing a reliable prediction of the actual response of the structure during an earthquake.

The purpose of analysis, however, is not to predict response but rather to provide information that an engineer can use to proportion members and to estimate whether or not the structure has sufficient stiffness to limit deformations and avoid overall instability. In short, the analysis only has to be "good enough for design." If, on the basis of any of the above analyses, the elements are properly designed for strength, the stiffness requirements are met and the elements and connections of the structure are detailed for inelastic response according to the requirements of the *Provisions*, the structure will likely survive an earthquake consistent with the maximum considered ground motion. The exception would be if a highly irregular structure were analyzed using the ELF procedure. Fortunately, the *Provisions* safeguards against this by requiring three-dimensional dynamic analysis for highly irregular structures.

For the structure analyzed in this example, the irregularities were probably not so extreme such that the ELF procedure would produce a "bad design." However, when computer programs (e.g., SAP2000 and ETABS) that can perform modal-response-spectrum analysis with only marginally increased effort over that required for ELF are available, the modal analysis should always be used for final design in lieu of ELF (even if ELF is allowed by the *Provisions*). As mentioned in the example, this does not negate the need or importance of ELF analysis because such an analysis is useful for preliminary design and components of the ELF analysis are necessary for application of accidental torsion.

The use of time-history analysis is limited when applied to a linear elastic model of the structure. The amount of additional effort required to select and scale the ground motions, perform the time-history analysis, scale the results, and determine envelope values for use in design is simply not warranted when compared to the effort required for modal-response-spectrum analysis. This might change in the future when "standard" suites of ground motions are developed and are made available to the earthquake engineering community. Also, significant improvement is needed in the software available for the preprocessing and particularly, for the post-processing of the huge amounts of information that arise from the analysis.

Scaling ground motions used for time-history analysis is also an issue. The *Provisions* requires that the selected motions be consistent with the magnitude, distance, and source mechanism of a maximum considered earthquake expected at the site. If the ground motions satisfy this criteria, then why scale at all? Distant earthquakes may have a lower peak acceleration but contain a frequency content that is more significant. Near-source earthquakes may display single damaging pulses. Scaling these two earthquakes to the *Provisions* spectrum seems to eliminate some of the most important characteristics of the ground motions. The fact that there is a degree of freedom in the *Provisions* scaling requirements compensates for this effect, but only for very knowledgeable users.

The main benefit of time-history analysis is in the nonlinear dynamic analysis of structures or in the analysis of non-proportionally damped linear systems. This type of analysis is the subject of Example 3.2.