SEISMIC BEHAVIOR OF

RETAINING WALLS

by

Gabriel Fernandez

University of Illinois

Urbana, Illinois

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SEISMIC BEHAVIOR OF GRAVITY RETAINING WALLS

ABSTRACT: Force-based design approaches have been widely used for seismic design of retaining walls ever since Mononobe-Okabe, M-O, introduced their equation in the late 1920's. Research conducted over the last thirty years has revealed that the actual conditions during earthquake shaking of retaining structures can be different from those assumed in the M-O equation. The difference becomes exacerbated for large ground accelerations in excess of 0.4 g. Furthermore, the adequate structural behavior observed in retaining walls under large seismic motions (avg. > 0.4 g) indicates that the actual loads on the wall structures are much lower than those anticipated by the M-O approach. Richards and Elms (1979) and Whitman and Liao (1985) developed a displacement-based design approach that allows limited permanent seismic-induced displacement of the wall. Lack of consideration of site specific ground motions can lead to over estimates of displacement in most situations. Also, unrealistic high wall displacements are estimated from the use of peak ground accelerations associated with frequency values that do not excite the critical soil wedges loading the wall. The analysis presented in this study indicates that overestimates of seismic-induced loads using the M-O approach result from the unrealistic large size soil wedges that are assumed to be thrusted against the retaining walls by the seismic motions. A new approach to estimate seismic-induced loads and/or yield acceleration of retaining walls taking into account the dynamic response of the backfill materials under specific ground motions is proposed in this analysis. The results obtained from this approach need to be calibrated with the field data obtained from observed displacements in retaining walls subjected to significant ground motions.

INTRODUCTION

An overview of the historical developments of the various approaches followed to evaluate and design retaining walls under seismic loading is presented first. In this regard, a review is made of the basis of the M-O equation to estimate seismic loads on retaining walls, followed by an assessment of the Richards and Elms (1979) and Whitman and Liao (1985) design approaches based on estimates of the seismic-induced wall displacements. An evaluation is then made of these design approaches identifying the critical assumptions that in our opinion result in the prediction of excessive seismic-induced loads and/or displacements in retaining wall design. Finally a detailed description is presented of the approach proposed in this study to estimate retaining walls seismic induced loads and displacements. Descriptions of limited observation on retaining wall behavior during earthquake loading is also presented to calibrate the new displacement-based design approach.

THE MONONOBE-OKABE EQUATION

The Mononobe and Matsua (1929) and Okabe (1926) analysis for dynamic lateral pressures is an extension of the Coulomb sliding wedge theory to account for inertia forces corresponding to horizontal and vertical accelerations, $k_h g$ and $k_v g$, acting on the sliding soil wedge behind the wall.

The method was developed for dry cohesionless materials assuming that the wall yields sufficiently to produce an active pressure condition; and that the maximum shear strength of the backfill materials is fully mobilized along the potential sliding surface. The critical soil wedge immediately behind the wall is assumed to behave as a sliding rigid body that applies the load on the wall. Ground accelerations are assumed to act uniformly throughout the soil wedge and earthquake forces are represented by inertia forces $k_h W_s$ and $k_v W_s$, where W_s is the weight of the soil wedge. Force equilibrium analysis of the wedge alone (Fig. 1) gives the basic M-O formula for total active load given by:

$$P_{AE} = \frac{1}{2} \gamma H^2 (1 - k_v) K_{AE} \qquad (1)$$

where γ is the unit weight of the backfill, H is the height of the wall, and K_{AE} is the active seismic coefficient. The coefficient, K_{AE}, is a function of the geometries of the wall and the backfill, the strength of the backfill, the friction angle along the backfill-wall interface, and the horizontal and vertical ground acceleration coefficients, k_h and k_v respectively. The values of K_{AE} can be calculated as:

$$K_{AE} = \frac{\cos^2(\varphi - \beta - \psi)}{\cos\psi \cos\beta (\delta + \beta + \psi) \quad 1 + } \qquad (2)$$

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$$[\quad \_ \cos(\delta + \beta + \psi \cos(i - \beta) \quad]$$

where

Φ	=	friction angle of the backfill materials
β	=	angle of inclination of the back of the wall (Figure 1)
i	=	angle of inclination of the backfill.
δ	=	friction angle at the backfill-wall interface; and

$$\psi = \tan^{-1} \left[\frac{k_h}{1 - k_v} \right] \qquad (3)$$

where

k_h	=	a_h/g;
a_h	=	horizontal ground acceleration
k_v	=	a_v/g; and
a_v	=	vertical ground acceleration

The estimates of K_{AE} in Eq. 2 are based on the development of a seismic-induced critical failure surface within the backfill materials, which has an angle of inclination, α_{AE}, with respect to the horizontal. The angle of inclination α_{AE} of the failure surface under seismic loading is given by the following formula:

$$-\text{tang}(\varphi - \psi - i) + C_{1E}$$

$$\alpha_{AE} \;=\; \varphi - \psi + \tan g^{-1} \left[\frac{\quad}{C_{2E}} \right] \qquad (4)$$

where C_{1E} and C_{2E} are functions of φ, θ, and i, as shown in Appendix 1. The other terms in Eq. 4 are the same as defined above.

The effect of the magnitude of the seismic-induced ground motions on the value of the seismic active coefficient, K_{AE}, as well as on the size of the critical sliding wedge is shown in Figures 2 and 3, respectively. As shown in Figure 2, for a given wall geometry and backfill materials as the horizontal seismic-induced ground acceleration, k_hg, increases the magnitude of the K_{AE} coefficient concurrently increases and reaches values of twice the static coefficient for a ground acceleration of 0.4 g. Simultaneously, as the horizontal ground acceleration increases, the inclination of the critical sliding surface (K_{AE}) with respect to the horizontal decreases, Fig. 3, resulting in an increasingly larger sliding wedge which is assumed to behave as a rigid body which experiences a uniform ground acceleration and is supported by the retaining wall. Under relative large ground accelerations, the critical sliding surface within the backfill materials becomes flat and the size of the sliding wedge becomes larger and exceeds the size of a “passive” wedge that develops under large stresses against the wall. Because of the compressible nature of the soil materials, large size soil wedges do not respond to the high frequency pulses associated with peak ground accelerations but are excited by lower acceleration pulses as will be shown later. In addition, flat wedges can result in dimensions larger than the wave length of the stress pulse, which indicates that such sliding mass would not be accelerated simultaneously, in the same direction. Thus, the actual seismic loads on the wall can be much lower than those anticipated by the M-O equation.

RICHARDS AND ELMS APPROACH (1979)

Richards and Elms (1979) made use of Newmark's (1965) sliding block model to establish a displacement based design approach for gravity retaining walls. Seismic-induced permanent wall displacements are estimated in a manner analogous to the Newmark (1965) sliding block procedure developed originally to estimate seismic slope stability. The weight of the wall is designed to develop a "yield" acceleration, a_y, that would result in tolerable wall displacements. The Richards and Elms (1979) approach to estimate the yield acceleration, a_y, is an iterative procedure that relies on the calculation of seismic-induced loads on the wall based on the M-O method. Therefore, under relatively large ground accelerations the loads on the wall are also based on the development of unrealistically large sliding wedges within the backfill materials. Expressions to estimate permanent wall displacements were proposed by Franklin and Chang (1977) based on the analysis of 169 horizontal and 10 vertical corrected accelerograms as well as several synthetic records.

Franklin and Chang (1977) used a progressive failure model in the calculation of standardized maximum displacements, by drawing envelope curves for different acceleration records (Fig. 4). The total displacement of a gravity wall due to an earthquake does not occur at once, but rather develops as a series of smaller incremental displacements. Incremental failure by base sliding is illustrated in the Fig. 5, which shows a sequence of graphs of the variation of the seismic-induced velocities and displacements of the wall and backfill materials with time during an earthquake. As the ground acceleration develops, the wall and ground move together from point 0 to point a, as shown in Figure 5. At point a, a yield acceleration ($k_{hy)=}$), is reached where sliding occurs along the base of the wall. Once wall sliding occurs, it is assumed that the wall displaces with a velocity lower than the seismic-induced velocity in the surrounding ground, which generates a relative displacement between the wall and the backfill materials. As the peak ground acceleration decreases, sliding along the base of the wall ceases and at point b, the ground and wall velocities become equal and the wall-backfill system once again moves as one rigid body. This situation continues until at point c, where the ground acceleration again exceeds the yield acceleration k_{hy} and the process is repeated. Thus, a relative velocity develops between wall and the backfill materials for a number of short intervals, during which relative wall displacements takes place in discrete steps.

The design approach proposed by Richard and Elms (1979) was based on the view that it would not be economical to design walls under seismic loading with no allowance for some displacement. The basic approach proposed by Richards and Elms (1979) to determine the magnitude of the seismic-induced displacement is presented below.

The force equilibrium in the free body-diagram for a retaining wall shown in Fig. 6, was established as:

$\Sigma F_{Vertical} = 0$

$$N = (1 - k_v) W_w + P_{AE} \sin(\delta + \beta) \quad (4)$$

where: W_w = weight of the wall; and
P_{AE} = the total force, static plus seismic applied by the soil wedge on the back of the wall.
δ = friction angle at the soil-wall interface.
N = Normal force on the base of the wall
β = angle of inclination of the back of the wall with respect to the vertical

$\Sigma F_{Horizontal} = 0$

$$F = k_h W_w + P_{AE} \cdot \cos(\delta + \beta) \quad (5)$$

For sliding along the base of the wall, the normal force on the base, N, and the frictional force, F, are related by Equation (6).

$$F = N \tan \varphi_b \quad (6)$$

where: φ_b = friction angle along the base of the wall.

The seismic-induced vertical acceleration was ignored and the yield acceleration, a_y, at which sliding takes place along the base of the wall was estimated as

$$a_y = \left[\tan\varphi_b + \frac{P_{AE} \sin(\delta + \beta) \tan\varphi_b - P_{AE} \cos(\delta + \beta)}{W} \right] g \quad (7)$$

Richard and Elms (1979) proposed the use of the design envelope in Figure 4, which is based on Franklin and Chang's standardized seismic-induced displacements to estimate the displacements of gravity walls. The Franklin and Chang's relationship is expressed as:

$$d = 0.087 \frac{V^2_{max}\, a^3_{max}}{a^4_y} \quad (8)$$

where: d = displacement of a gravity wall in inches;

V_{max} = peak ground velocity of the design earthquake in in./s;

a_y = $k_{hy} \cdot g$ = the yield acceleration that results in sliding along the base of the wall in in./s^2; and

a_{max} = peak horizontal ground acceleration in in./s^2.

On the basis of the analysis presented above, Richard and Elms (1979) have suggested the following design procedure.

1. Decide upon an allowable maximum wall displacement, d. This is a key step of the displacement-based design. Allowable displacements should not cause an irreversible level of damage.

2. Use Eq. 8 to obtain the value of the yield acceleration, k_{hy}, corresponding to the allowed displacement chosen, d.

$$k_{hy} = \left(\frac{0.087\, V^2\, a^3_{max}}{d} \right)^{1/4}$$

3) Use the estimated yield acceleration, k_{hy}, in Eqs. 1 and 7 to obtain the required wall weight, W_w.

4) Apply a suitable factor of safety to the wall weight.

It is important to note that the estimated seismic-induced wall displacements rely on the validity of the M-O assumptions to assess the yield acceleration, a_y, and that the Richards and Elms (1979) method considers only sliding but not tilting of the wall. Both of these assumptions are questionable and result in anticipated displacements larger than those observed seismic loads.

WHITMAN AND LIAO DESIGN APPROACH (1985)

Whitman and Liao (1985) identified several modeling errors that result from the simplifying assumptions of the Richards-Elms procedure. The most significant are the neglect of the dynamic response of the backfill, the lack of tilting mechanisms, and the omission of vertical accelerations. The dynamic response of the backfill can have a significant influence on wall displacements (Nadim, 1982). Amplification occurs when input motions coincide with the natural period of the backfill and produce considerably different permanent displacement than the rigid-block model used by Richards and Elms. Analyses in which the backfill wedge and wall were treated as separate blocks (Zarrabi-Kashani, 1979) show that the kinematical requirements of horizontal and vertical displacement of the backfill wedge cause systematically smaller displacements than the single-block model of Richards and Elms. Studies of combined tilting and sliding (Nadim, 1980; Siddharthan et al., 1992), indicate that tilting mechanisms generally increase wall displacements over those produced by sliding-only models such as that of Richards and Elms. Consideration of vertical accelerations produces slightly larger displacements than when they are neglected. Whitman and Liao quantified and combined the effects of each of these sources of modeling error.

Using the results of sliding block analyses of 14 ground motions by Wong (1982), Whitman and Liao found that the permanent displacements were lognormally distributed with a mean value equal to

$$d_R = \frac{37\, V^2_{max}}{a_{max}} \exp\left(-9.4 \frac{a_y}{a_{max}}\right) \qquad (9)$$

Again, the seismic-induced wall displacements in Eq. 9 rely on the validity of the M-O assumptions to estimate the yield acceleration coefficient, K_{hy}.

UNCERTAINTIES IN THE CURRENT PREDICTION OF RETAINING WALL BEHAVIOR UNDER SEISMIC LOADS

Previous studies by Whitman (1990) have indicated that failure or excessive movement of gravity retaining walls has occurred rather infrequently, even during major earthquakes, although quay walls should be excluded from this generalization. Whitman (1990) concluded that this “high performance” might be attributed to conservative assumptions made during the design stage.

The observed performance of a 70 ft high retaining wall in the Koyna earthquake of December 11, 1967 also indicates that the actual behavior of retaining walls under strong seismic loading is far better than that anticipated by current evaluation methods. The 70 ft high wall with a 2:1 (V:H) batter at the spillway basin of the Koyna dam escaped damage during the earthquake on December 11, 1967. The peak ground acceleration was 0.63 g and the peak ground velocity of 22 cm per sec. There was no sign of either overturning or even overstressing of the wall or the foundation soil. Based on an analysis of the wall using the conventional procedures and the soil properties used in the original design computations, Krishna, et al (1969) namely 1) M-O theory is valid, 2) the inertia force on the wall is given by the product of the weight of the wall and the seismic coefficient, 3) the dynamic increment acts at 2/3 the height of the wall above its base, and 4) the vertical inertia force acts in the worst possible direction from stability consideration, concluded that a) the wall will overturn at a seismic coefficient of 0.32 and overstressing of the foundation will occur at a seismic coefficient of 0.26. If the dynamic increment is assumed at mid-height, overturning occurs at 0.53 and overstressing at about 0.42. Overstressing does not occur up to 0.5 if the dynamic increment acts at 0.45 h or below. Given the lack of damage experienced by the wall, it can be concluded that current evaluation methods overestimate the effect of seismic loading on this retaining wall.

Whitman (1990) summarized uncertainties and errors in prediction of seismic behavior of retaining walls to the following aspects:

1) Unpredictable details of seismic-induced ground motion
(Frequency content, duration, directionality, vertical motions);

2) Uncertain in the resistance parameters of the backfill materials
(Friction angle of backfill, base of wall, and backfill-wall interface); and

3) Model errors

Other shortcomings of current evaluation and design methods to assess retaining wall behavior under seismic loading are discussed below.

The M-O approach is based on a force equilibrium analysis that determines the geometry of the soil wedge for the combined static and seismic forces, P_{AE}, on the wall. Under static loading, the maximum load on the wall corresponds to the active wedge geometry determined by well-known methods (Coulomb or Rankine). Under the M-O approach as the seismic acceleration increases, the size of the critical wedge (maximum wall load) also increases because the inclination of the critical sliding surface becomes flatter. The magnitude of the wall load, the inclination of the failure surface (with respect to the horizontal), and the size of the critical soil wedge that develops in horizontal backfill materials behind a 20 m high wall for different peak ground accelerations are summarized in Table 1. A friction angle, φ, of 35° was assumed for the backfill materials behind the wall, a friction angle δ of 25° was assumed along the wall-backfill interface, and the back of the wall was assumed to make an angle β = 25° with the vertical.

As indicated in Table 1, for a ground acceleration of 0.5 g the length, L, of the critical wedge is about 2.75 the height of the wall, which is equal or larger than the size of the failure wedge corresponding to a “passive condition”. At slightly larger acceleration levels shallow failures begin to occur within the soil materials behind the wall, limiting the maximum load that can be transferred to the wall.

There are also additional constrains that can limit the magnitude of seismic-induced loads on retaining walls to values lower than those estimated from the M-O method. As indicated in Table 1, for peak ground accelerations between 0.4 g and 0.5 g, the size of the critical wedge is such that its lateral extent behind the wall is about 2 times the height of the wall. In many instances (particularly with retaining walls adjacent to rock cuts), the backfill behind the wall is limited and the M-O wedge cannot develop within the backfill. In such cases, the seismic loads computed on the basis of the M-O critical wedges are unrealistic.

TABLE 1

CASE NO.	a_h HORIZONTAL PEAK GROUND ACCELERATION	TOTAL LOAD ON THE WALL Tons/M	INCLINATION OF WEDGE FAILURE SURFACE WITH HORIZONTAL, degrees	CRITICAL WEDGE GEOMETRY L/H*
1	0.0 g	240	70	0.36
2	0.1 g	280	60	0.58
4	0.3 g	425	42	1.1
5	0.4 g	550	32	1.6
6	0.5 g	780	20	2.74
7	0.6 g	1300	12	4.7
8	0.65 g	3660	2	29.0

* L is the horizontal distance from the toe of the wall to the tail of the critical soil wedge.
H is the height of the wall.

Finally, as the size of the wedge becomes larger, its natural frequency of vibration decreases, and thus it is not likely to respond to peak ground accelerations that are usually associated with high frequency values. Large damping ratios ($\beta > 10\%$) associated with backfill materials also result in ground motion amplifications that are likely to be nil. Thus, the maximum acceleration uniformly applied to relatively large soil wedges computed by the M-O method might be significantly lower than the peak ground acceleration. This phenomenon results in reduced seismic loads on the wall. An assessment needs to be made of the “effective” ground acceleration corresponding to various size soil wedges behind the wall. In general, the smaller the wedge the higher its natural frequency of vibration, and thus the larger the “effective” ground acceleration applied.

The approach proposed by Richard & Helms (1979) as well as Whitman & Liao (1985) recognize the potential for slippage to take place along the base of the wall once the load imposed by the soil wedge overcomes the frictional resistance along the base. The wall can then be seized to maintain the seismic-induced displacements within tolerable values.

If the displacements are found to be excessive, design modifications (i.e. increase the weight of the wall) can be implemented to reduce their magnitude to a tolerable value.

Although the methodologies described above are an improvement from the M-O approach, field observations indicate that both approaches anticipate retaining wall movements far in excess of those experienced during major earthquakes. In our opinion the main shortcomings of these two methodologies can be summarized as follows:

1) Estimates of the yield accelerations that cause sliding along the base still rely on the validity of the basic assumptions made by the M-O method to estimate the seismic-induced loads on the wall. As indicated above the seismic-induced design accelerations depends on the size of the critical soil wedge behind the wall. Effective accelerations instead of peak ground accelerations need to be used to determine the magnitude of the seismic load in the wall.

2) In both methodologies Richard and Helms as well as Whitman and Liao, the specific ground motions expected at the site of interest are not used to estimate wall displacements. Instead, an envelope of seismic-induced displacements estimated from ground motions triggered by earthquakes with a wide range of magnitudes is used. The use of a database of worldwide records has the advantage of including some particularly 'damaging' earthquakes of long duration and significant number of cycles. However, this approach may be grossly overconservative for less damaging earthquakes. In the case of lower-magnitude, near-site, earthquakes, slide-displacement that can accumulate is limited by the small number of significant cycles exceeding the yield acceleration. Because the number of cycles producing displacements is related to the magnitude of the earthquake, adjustments need to be made to take this factor into account.

3) Soil-increased strength due to high frequency loading may also have an effect on the wall loads. This factor can be taken into account by including the rate dependent increase of soil strength if any.

4) Both methodologies R&E and W&L do not take into account the potential for wall tilting triggered by the seismic induced loads. In soft-type foundations this potential might be more likely to occur than the displacement along the base. Wall tilting would also limit the maximum seismic-induced load in the wall.

PROPOSED APPROACH FOR SEISMIC DESIGN OF RETAINING WALLS

The proposed approach for seismic design of retaining walls is based on four basic tenets:

1) Retaining walls are designed to undergo some displacement during earthquake loading. The magnitude of the seismic-induced displacements should be maintained within tolerable values

2) Retaining walls are generally designed to accommodate static loads with a factor of safety of about 1.5 against sliding along the base, and a factor of safety of 3.0 against current overturning. The static design criteria can be the basis to establish the "yield" acceleration, a_y, required to trigger sliding and/or overturning of the retaining wall.

3) The dynamic response of the backfill should be taken into account to assess the magnitude of the seismic-induced accelerations on the soil wedge-retaining wall system; and

4) Seismic-induced wall displacements can be estimated on the basis of the Newmark's 1965 procedure to assess seismic slope stability.

Estimate of Yield Acceleration -- Retaining walls are designed to accommodate estimated static loads with a factor of safety of about 1.5 against sliding along the base, and a factor of safety of 3.0 against moment overturning. If the sliding mode of failure is evaluated, the equilibrium of forces on the wall under dynamic conditions, assuming the seismic-induced vertical acceleration to be nil, can be established as follows:

$$\Sigma F_{Vertical} = 0 \tag{10}$$

$$N = W_w + (P + \Delta P_{AE}) \sin(\delta + \beta) \tag{11}$$

where

P = static load on wall (Coulomb)

ΔP_{AE} = seismic-induced load wall increment

$$\Sigma F_{Horizontal} = 0 \tag{12}$$

$$F = (P + \Delta P_{AE}) . \cos(\delta + \beta) + K_{hy} W_w \tag{13}$$

where

K_{hy} = yield acceleration coefficient

For sliding along the base, $F = N \tan \varphi_b$ (14)

$$[W_w + (P + \Delta P_{AE}) \sin(\delta + \beta)] \tan \varphi_b = (P + \Delta P_{AE}) \cos(\delta + \beta) + K_{hy} W_w \tag{15}$$

Under static conditions with a factor of safety of 1.5 against sliding along the base of the wall, the summation of horizontal forces on the wall yields

$$1.5 = \frac{[W_w + P \sin(\delta + \beta)] \ \text{tang}\ \varphi_b}{P \cos(\delta + \beta)} \tag{16}$$

Equation (16) indicates that the reserve friction force, F_s, along the base of the wall available to accommodate seismic-induced loads can be estimated as:

$$F_s = 1/3\ [W_w + P \sin(\delta + \beta)] \tan \varphi_b \quad (17)$$

The seismic-induced horizontal load generated by the inertia force, F_I, of the wall and the critical soil wedge behind the wall can be evaluated as:

$$F_I = K_{hy} W_w + W_s K_{hy} = W_s \tan (\varphi - \alpha_{AE}) \quad (18)$$

where

W_s = is the weight of the critical soil wedge behind the wall under seismic loading.

α_{AE} = cycle of inclination of the base of the critical sliding surface with respect to the horizontal.

Summation of horizontal forces results in

$$F_s = F_I \quad (19)$$

then

$$K_{hy} W_w + W_s K_{hy} - W_s \tan (\varphi - \alpha_{AE}) = 1/3\ [W_w + P \sin (\delta + \beta)] \tan \varphi_b \quad (20)$$

Because the size of the critical wedge, (W_s, α_{AE}) depends on the magnitude of the acceleration applied to the wedge the solution of Eq. 20 requires an iterative process where a value of K_{hy} is chosen and the corresponding size of the wedge, W_s and α_{AE}, are estimated from Eq. 20. The magnitude of W_s and α_{AE} are mutually dependent. The, α_{AE}, estimate from Equation 20 needs to be compared with the size of the wedge resulting from the solution of Equation 4. If the two values of α_{AE} coincide the chosen value of K_{hy} is correct. If not the process needs to be repeated again.

Estimate of seismic-induced accelerations on soil wedge – The dynamic response of the sliding wedge-wall system depends on the geometry and the stiffness of the backfill materials and the wall as well as on the amplitude and frequency content of the ground motions. Nadim and Whitman (1983) have demonstrated that the seismically induced sliding displacement of retaining walls is affected by the ratio of the predominant frequency of the earthquake ground motion to the frequency of the backfill (i.e., f/f_1). Thus, an 'effective accelerations' instead of the peak ground acceleration needs to be used to determine the seismic load on the wall. The "effective acceleration" of the soil-wedge retaining wall system can be estimated taking into account the ratio of the predominant frequency of the earthquake ground motion to the fundamental frequency of the wedge and wall components.

A tentative approach to assess the fundamental period of vibration of the soil wedge behind the wall is to assume that the critical soil wedge under seismic loading behaves as a single degree of freedom system with a fundamental frequency of vibration given by:

w = √k,_/,_m,_ (21)

where m = mass of the seismic-induced sliding wedge

k = compressional stiffness of the backfill materials within the sliding wedge

The compressional stiffness of the backfill wedge under a uniform horizontal acceleration equal to a_h can be estimated as

$$k = \frac{F_h}{\delta} = \frac{m \cdot a_h}{\frac{1}{2}\,\frac{\gamma_s}{E_m}\,\frac{a_h}{g}\,L^2}$$

and thus the fundamental frequency of vibration can be established as:

$$w = \frac{2 E_m g}{\gamma_s L^2}$$

where: E_m = modulus of the backfill materials

γ_s = unit weight of the backfill materials

g = acceleration of gravity and

L = maximum wedge dimension

For the example in Table 1, at a peak ground acceleration of 0.5 g the estimated length of the "critical" wedge is given as:

$$L = 2.74 \times 20 \text{ m} = 55 \text{ m} = 180 \text{ ft}$$

If the modulus of the backfill materials, E_m, is equal to 3000 psi = 432,000 psf

γ_s = 125 pcf

then

$$w = \frac{2 \times 432{,}000 \text{ psf} \times 32.2 \text{ ft/sec}^2}{125 \text{ lb/ft}^3 \times (180)^2 \text{ ft}^2} = 2.6 \text{ radians/sec}$$

$$F = \frac{w}{2\pi} = \frac{2.6}{2 \times 3.14} = 0.41 \text{ cycles/second}$$

Thus the fundamental frequency, F, of the soil wedge is equal to 0.41 Hertz.

For most seismic ground motions the 0.4 Hertz frequency corresponds to the velocity-bound region of the spectra, where the effective accelerations are significantly smaller than the peak ground motions. Thus, the "effective" acceleration to be used in retaining wall design is likely to be only a fraction of the peak ground acceleration which must be modified for the geometry and damping of the critical soil wedge considered. An estimate of this acceleration can be obtained by an iterative process, where an effective acceleration equal to a fraction (1/2) of the peak ground acceleration is initially chosen, and the size of the corresponding critical soil wedge behind the wall is estimated. The fundamental frequency, F, of the estimated soil wedge is then determined, and entered in a plot of the design spectra to assess the seismic-induced acceleration likely to be experienced by the soil wedge. If the spectral acceleration is equal to the effective acceleration initially chosen to determine the size of the soil wedge the iterative process is completed. If not, a different initial acceleration is chosen and the process is repeated again.

Estimate of seismic-induced wall displacement – Once the yield acceleration coefficient K_{hy} and the effective ground acceleration A, are determined, the corresponding seismic-induced wall displacements can be estimated by using Newark's (1965) approach.

$$\delta = \frac{V^2_{max}}{2\ gN} (1 - N/A)\ A/N \qquad (22)$$

where V_{max} = peak particle velocity

N = K_{hy} g = yield acceleration

A = "equivalent" ground acceleration

δ = seismic-induced wall displacements

CONCLUSIONS AND RECOMMENDATIONS

Permanent wall displacements estimated using the proposed approach could be calibrated with field data, from gravity retaining walls that have sustained significant ground motions generated by earthquakes of relatively large magnitude.

On the basis of the evaluation presented in this study, including the observed behavior of retaining walls under significant seismic loading it can be concluded that:

1) Retaining walls with an adequate static factor of safety, of 1.5 or above, are capable of sustaining significant seismic loading without excessive damage;

2) The static factor of safety can be increased in areas of intense seismic activity;

3) Free-draining well compacted granular materials are recommended to be placed behind the wall for a distance at least equal to the height of the wall;

4) Generous drainage should be installed to minimize the development of water pressures behind the wall.

NOTATION

a_h	=	horizontal ground acceleration;
a_v	=	vertical ground acceleration;
a_{max}	=	peak ground acceleration;
B	=	the reaction at the base with components F and N;
d	=	displacement of wall (inches);
H	=	height of wall;
h_a	=	height of active soil force resultant;
i	=	slope of the ground surface behind the wall;
k_h	=	horizontal backfill acceleration coefficient;
k_v	=	vertical backfill acceleration coefficient;
k_{hy}	=	the coefficient of horizontal acceleration at which sliding starts;
V_{max}	=	peak ground velocity;
W_s	=	the weight of the wedge;

W_w	=	the weight of the wall;
β	=	slope of the wall to the vertical;
δ	=	angle of friction between the wall and soil;
φ'	=	angle of friction of soil;
φ_b	=	the friction angle at the base of the wall;
γ	=	unit weight of the soil;
θ	=	seismic angle;
ρ	=	the angle of inclination of the failure plane to the horizontal;
N	=	normal force at the base of the wall
F	=	friction force along the base of the wall

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Appendix I

C_{1E} =

$\sqrt{\tan(\varphi - \psi - i)\,[\tan(\varphi - \psi - i) + \cot(\varphi - \psi - \beta)]\,[1 + \tan(\delta + \psi + \beta)\cot(\varphi - \psi - \beta)]}$

$C_{2E} = 1 + \{\tan(\delta + \psi + \beta)\,[\tan(\varphi - \psi - i) + \cot(\varphi - \psi - \beta)]\,\}$

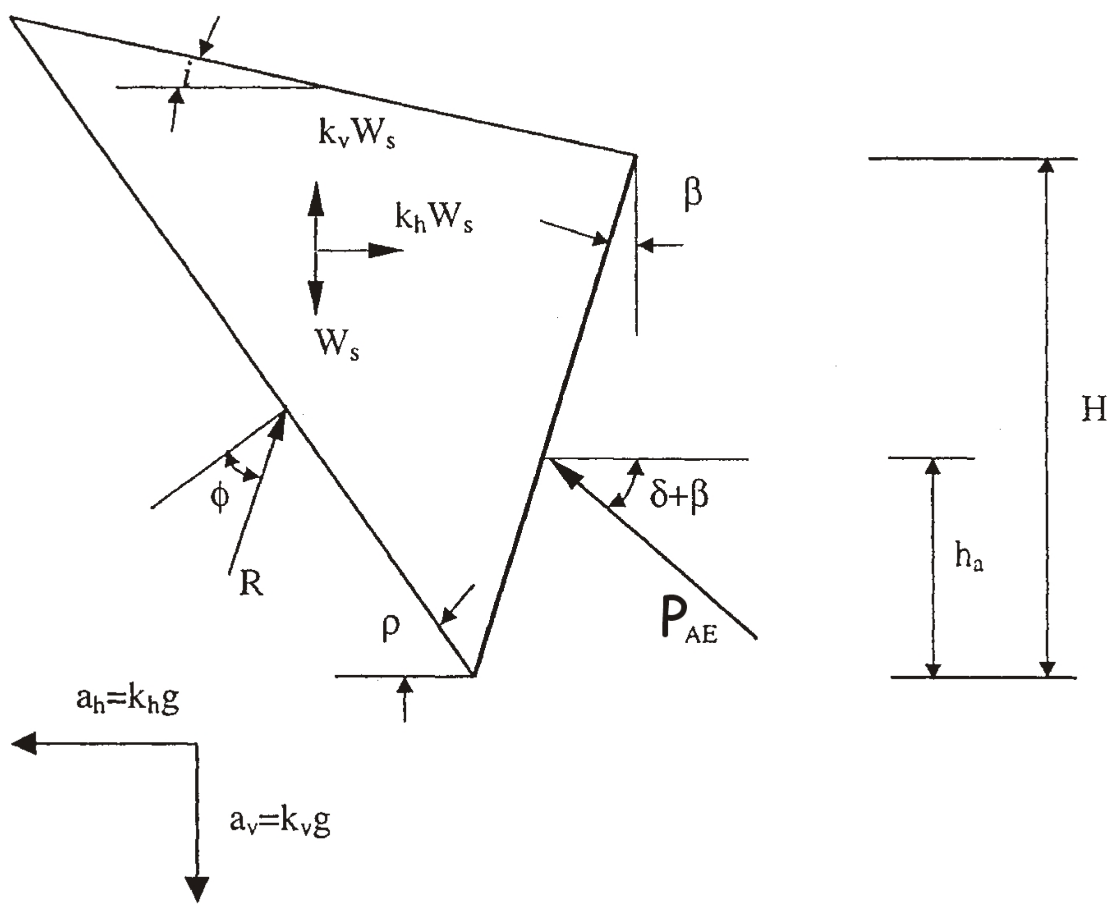


FIG. 1 Active Wedge and Force Polygon

Figure 2 Variation of Seismic Coefficient KAE with the Horizontal Acceleration Coefficent

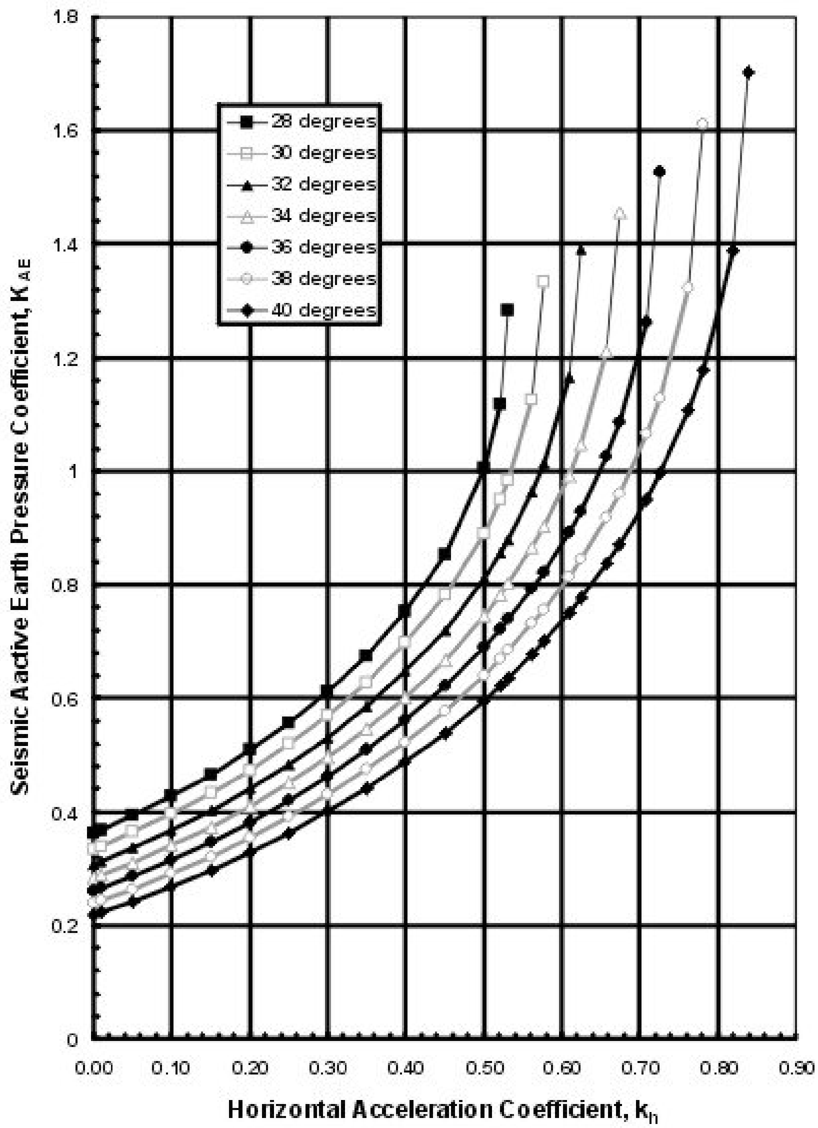
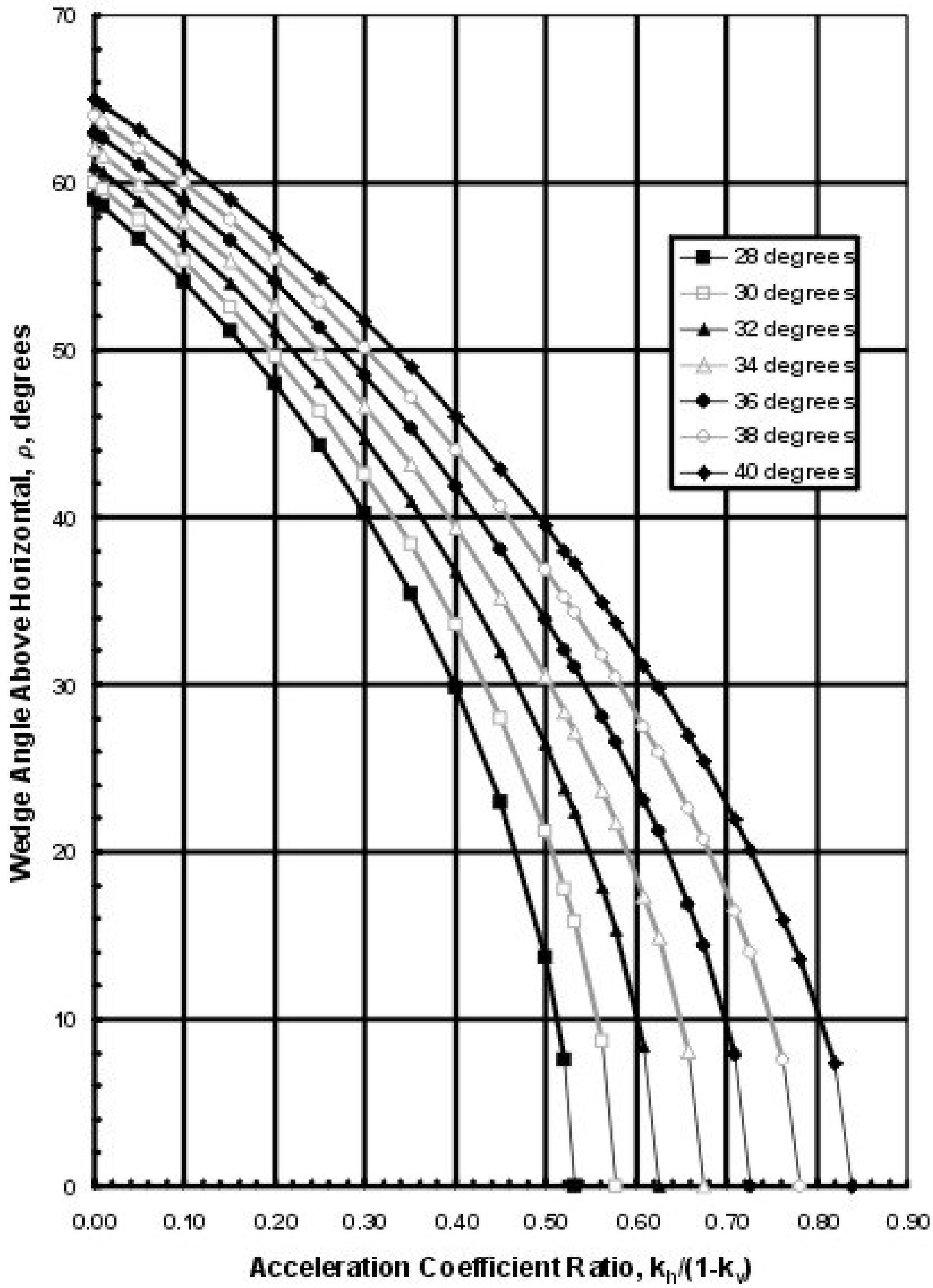


Figure 3 Variation of Wedge Angle with Acceleration Coefficent Ratio



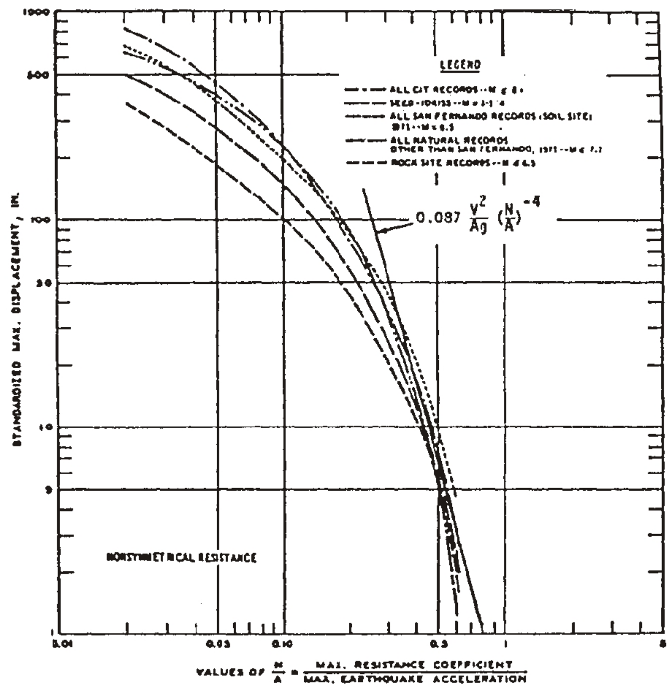


FIG. 4 Upper Bound Envelope Curves of Permanent Displacements for All Natural and Synthetic Records Analyzed by Franklin and Chang (after Richards and Elms, 1979)

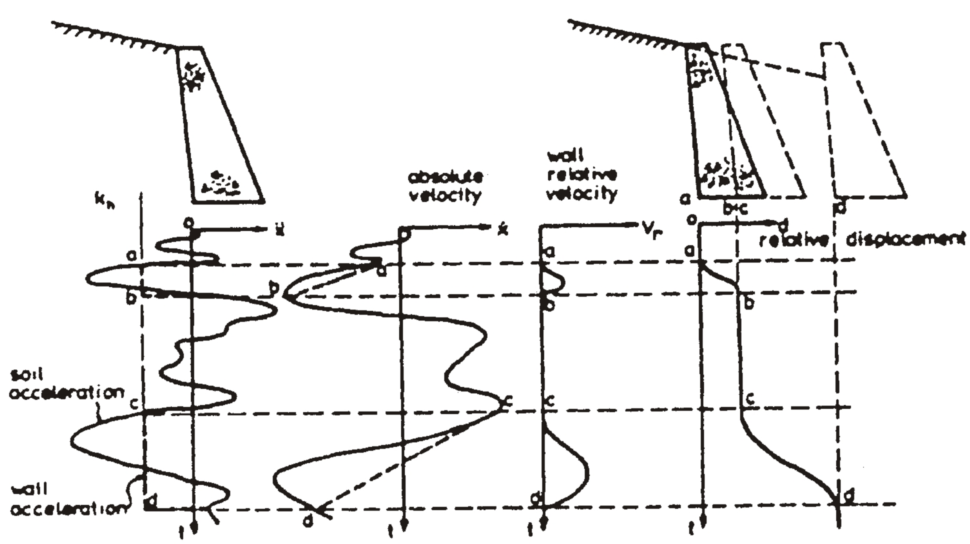


FIG. 5 Incremental Failure by Base Sliding (after Richards and Elms, 1979)

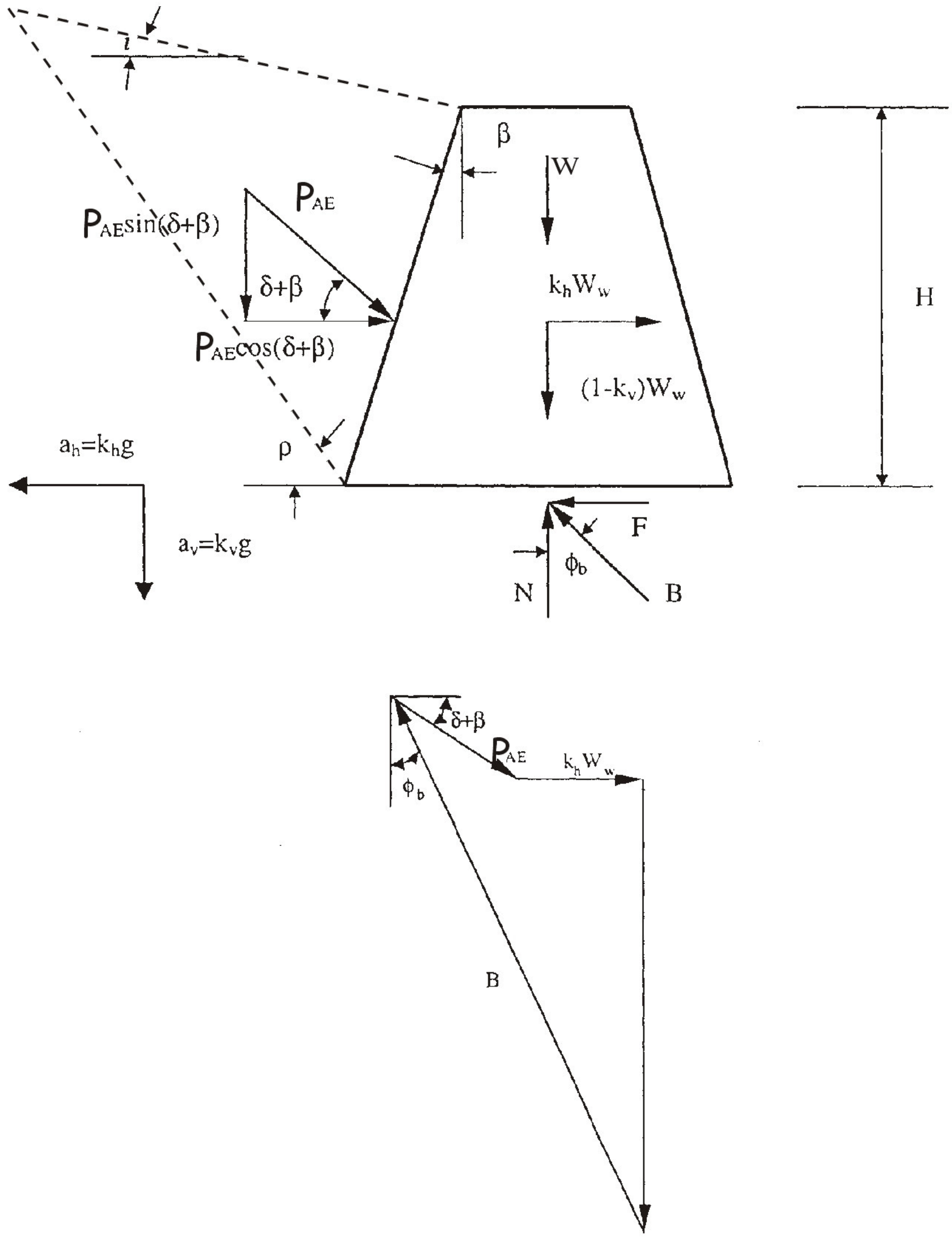


FIG. 6 Combined Force Diagram and Force Polygon: Active Case (All Quantities Shown Positive)